AN EMPIRICAL ANALYSIS OF THE DISTRIBUTION OF THE DURATIONS OF OVERSHOOTS IN A STATIONARY GAUSSIAN STOCHASTIC PROCESS

by

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ABSTRACT

This is an empirical analysis of the distribution of the durations of overshoots in a stationary gaussian stochastic process. The problem arose as a result of the National Aeronautics and Space Administration's need for a distribution formula for the duration time that certain atmospheric variables, such as wind speed or ambient temperature, exceed a specified level during a time interval of arbitrary length.

This analysis utilizes computer simulation and statistical estimation. Realizations of stationary gaussian stochastic processes with selected autocorrelation functions are computer simulated. Analysis of the simulated data revealed that the mean and the variance of a process were functionally dependent upon the autocorrelation parameter and crossing level. Using predicted values for the mean and standard deviation, by the method of moments, the distribution parameters could be estimated. Thus, given the autocorrelation parameter, crossing level, mean, and standard deviation of a process, the probability of exceeding the crossing level for a particular length of time could be calculated.

CHAPTER 1

INTRODUCTION

The purpose of this thesis is to report the results of an empirical analysis of the duration of overshoots above an arbitrary level in a stationary gaussian stochastic process. The study was conducted for the Terrestrial Environment Branch, Aerospace Environment Division, Aero-Astrodynamics Laboratory, George C. Marshall Space Flight Center, Alabama and was supported under NASA contract no. NAS8-29286. The results presented in this thesis are applicable in the prediction of extreme properties of such natural processes as wind speed, ambient temperature, and sea state.

1.1 Statement of the Problem

The problem dealt with by this thesis concerns the frequency distribution of the duration of overshoots above an arbitrary level in a gaussian stochastic process with an exponential autocorrelation function. A gaussian stochastic process is a stochastic process which has a gaussian distribution at any point in time. Such a process is stationary if it has a mean independent of time and an autocorrelation function dependent only on the distance between successive time points.

The problem is of general theoretical interest but numerical results are sparse. Rice (1945) did the fundamental work in studying both the crossing frequency and duration.

Favreau, Low, and Pfeffer (1956) studied the duration problem and hypothesized the negative exponential density for duration time--this hypothesis was disproved by Longuet-Higgins (1962). The recent texts by Kramer and Leadbetter (1967) and Kuznetsov (1965) give excellent summaries of the work in this area. To the author's knowledge this is the first investigation of duration times that is based on an extensive simulated data set.

1.2 Organization of the Analysis

Chapter 2 contains a description of the development of the simulation model and its underlying assumptions. In Chapter 3, the analysis of the model and our conclusions are presented. Chapter 4 contains a discussion of the application of the results of this analysis to specific problems concerning atmospheric variables.

CHAPTER 2

Simulation Model

Our first objective was to design a mathematical model of a stationary gaussian stochastic process with an exponential autocorrelation function. To do this, we made the following assumptions:

1) The process had a multivariate normal distribution.
2) The autocorrelation function, $R(\tau)$, was exponential, i.e., $R(\tau) = \exp(-\beta|\tau|)$.
3) The process was stationary, i.e., $R(t_i, t_j) = R(\tau)$, where $\tau = |t_i - t_j|$.
4) The expected value of a random variable X at time t was zero, i.e., $E[X(t)] = 0$.
5) The variance covariance matrix Σ was symmetrical and positive definite.

In the remainder of this chapter, we shall use $X(t)$ to denote a stochastic process satisfying the above conditions.

The process was considered over a time interval of [0,99], each realization consisting of 100 equally spaced points in the interval. In the case of a specific application, the interval of interest may be [0,T]. $X(t)$ would then be sampled at 100 equally spaced points in the interval. These points would be $t_0, t_1, \ldots, t_{99}$, where $t_i = (\frac{i}{100})T$, $i = 0,1,\ldots 99$. The autocorrelation parameter β would also require modification.

Odell (1971) presents the method of simulation used here. A summary of that technique follows.

Let $\underline{X}$ denote the vector $[X(t_0), X(t_1), \ldots, X(t_{99})]'$, where $'$ denotes matrix transposition. Let $\Sigma = (\sigma_{ij}) = E(X \cdot X')$, where E is the expectation operator. Σ is, of course, the variance covariance matrix. Thus for each element of Σ, σ_{ij}, $0 \leq i, j \leq 99$, we have

$\sigma_{ij} = E[X(t_i) \cdot X(t_j)] = R(t_i, t_j) = R(\tau)$, where $\tau = |t_i - t_j|$.

Hence we see that

$$\Sigma = \begin{bmatrix} R(0) & R(1) & R(2) & \cdots & R(99) \\ R(1) & R(0) & R(1) & \cdots & R(98) \\ R(2) & R(1) & R(0) & \cdots & R(97) \\ \vdots & \vdots & \vdots & & \vdots \\ R(99) & R(1) & & \cdots & R(0) \end{bmatrix}.$$

By assumption, $\underline{X}$ satisfies a multivariate normal distribution with mean $\underline{\mu} = \underline{0}$ and covariance matrix Σ. We write $\underline{X} \sim N(\underline{\mu}, \Sigma)$.

From Odell (1971, pg. 37) we have the following:

Theorem: If the 100 x 1 vector $\underline{Y} \sim N(\mu, \Sigma)$,

$\underline{\gamma}$ is a fixed 100 x 1 vector, and

$\underline{V} = A\underline{Y} + \underline{\gamma}$, then

$\underline{V} \sim N(A\underline{\mu} + \underline{\gamma}, A \Sigma A')$.

We generated the vector $\underline{Y} \sim N(\underline{0}, I)$, where I is the identity matrix. We factored the covariance matrix Σ into AA' by the Crout method. Thus, if $\underline{V} = A\underline{Y}$, it follows that $\underline{V} \sim N(\underline{0}, \Sigma)$. The vector $\underline{V}$ constituted a realization of the process $X(t)$. The vector $\underline{Y} = (y_0, y_1, \ldots, y_{99})$ was formed by generating a sequence of 100 independent standard normal variates. Hamming (1962) provides the technique for generating $\underline{Y}$. This method is summarized below.

An approximation to normally distributed random numbers may be obtained from a sequence x_k, k = 1, 2,...,K of uniformly distributed random numbers by the formula

$$y_i = \frac{\sum_{k=1}^{K} x_k - \frac{K}{2}}{\sqrt{K/12}}.$$ To utilize this method, the value of K was fixed at 12, reducing the formula to $y_i = \sum_{k=1}^{12} x_k - 6$. Thus, we were able to produce a sequence $\underline{Y} = (y_0, y_1, \ldots, y_{99})$ of standard normal variates with mean 0 and unit variance. Hence, we had $\underline{Y} \sim N(\underline{0}, I)$.

For each autocorrelation function, we generated 250 realizations which required 250 random vectors $\underline{Y}_i$, i=1,...,250, each of which required 100 standard normal variates. Each normal variate required 12 uniformly distributed random numbers. Hence, for each autocorrelation function, 250 x 100 x 12 = 300,000 uniformly distributed random numbers were required. The power residual method was used to generate these random numbers, each one being normalized to (0,1). That is,

$$r_n = \text{normalized } (s_n), \text{ where } s_n = \lambda \cdot r_{n-1}.$$

The computer used for the generation was a Univac Series 70/46 which has an integer capacity of $2^{31} - 1$.

In order to transform each vector $\underline{Y}_i$ into a realization $\underline{V}_i$, via $\underline{V}_i = A\,\underline{Y}_i$, it was necessary to factor Σ into $A \cdot A'$. Since Σ is a symmetrical, positive definite matrix, it can be factored into a lower triangular matrix and its transpose.

The factorization was accomplished using the Crout factorization method, Odell (1971, pg. 38). This method is summarized in Carter and Madison (1973).

The autocorrelation function $R(\tau) = \exp(-\beta|\tau|)$ determines the degree of association between pairs of values of $X(t)$. We simulated the process for twelve autocorrelation functions by assigning β, the autocorrelation parameter, the values .005, .010, .025, .050, .075, .10, .25, .50, .75, 1.0, 2.5, 5.0. With $\beta = .005$, the process had a correlation above .61 throughout and .99 for successive pairs of points. For $\beta = 5.0$, $R(2) = .00004539$, meaning that the values were essentially independent after two time intervals. The range of β values used proved to be appropriate for analysis purposes.

CHAPTER 3

Data Analysis

With the data sets generated, as described in Chapter 2, the first problem was that of determining the durations of overshoots above selected levels. From Sveshnikov (1966), we see that in a continuous random process $X(t)$, $X:D \rightarrow R$, two conditions must be met in order for an overshoot above a level a to occur in the interval $(t, t + dt)$. They are that at a given time t we have $X(t) < a$, while $X(t + dt) > a$. We say that an overshoot occurs between t_i and t_j provided $X(t_i) \leq a$, $X(t_j) \leq a$, and $X(t) > a$ for all $t \varepsilon (t_i, t_j) \cap D$. In the discrete case, we see that the two conditions required for an overshoot to occur above a level A are that $X(t_n) \leq A$, while $X(t_{n+1}) > A$, some n. We say that an overshoot occurs between t_n and t_m provided $X(t_n) \leq A$, $X(t_m) \leq A$, and $X(t_i) > A$ for $n < i < m$. Figures 1 and 2 illustrate an overshoot in the continuous case and one in the discrete case, respectively.

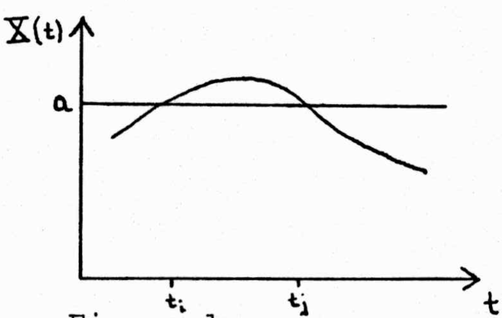


Figure 1.

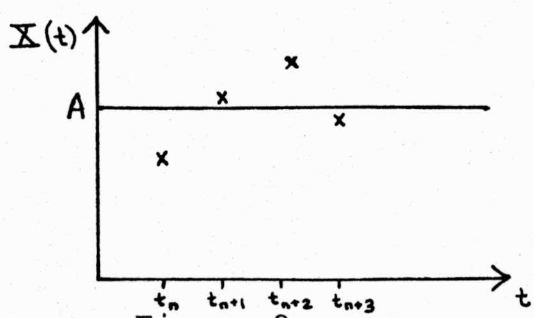


Figure 2.

In the continuous case, the duration of the overshoot between t_i and t_j would clearly be $(t_j - t_i)$. In the discrete case, to determine the duration we can only count the number of points t_i between t_n and t_m such that $X(t_i) > A$, while $X(t_n) \leq A$, and $X(t_m) \leq A$, which is $m - n - 1$.

Each realization in our data set consisted of 100 points $X(t_0)$, $X(t_1), \ldots, X(t_{99})$. So for a given level A, we located each overshoot, that is we found i such that $X(t_{i-1}) \leq A$, while $X(t_i) > A$, and then counted the points until $X(t_j) \leq A$; the duration was $j - i$. $(0 < i < j \leq 99)$. The question soon surfaced of whether to count as overshoots those points that began above A, $X(t_0) > A$, or which ended above A, $X(t_{99}) > A$. Noting that for small β values the system has a lot of memory, if the process began with $X(t_0) > A$, it was probable that it would stay above A for a long time. Similarly, if the process exceeded A somewhere between t_0 and t_{99}, it was likely to stay above A up to and including t_{99}. For larger β values, we did not feel that counting these so called T0 and T99 overshoots would appreciably affect our results. We therefore decided to include these types of overshoots in our counting.

We counted overshoots above levels A = 0, .5, .75, 1, 1.25, 1.5, 1.75, 2. 2.5, 3, 3.5, 4. Recalling that the process has a mean 0 and unit variance, we were equivalently considering overshoots above the mean, .5 standard deviations above the mean, .75 standard deviations above the mean, etc. In applications, for a process with mean μ and variance σ^2, the overshoots above level A would be equivalent to overshoots above the value of $A \cdot \sigma + \mu$.

The value 3.0 was selected as the upper limit of the A levels for analysis purposes. In the independent case, only .13% of the values would be above 3.0, and in the more correlated cases, the number of points above 3.0 and thus, the number of overshoots, would likely decrease. Zero was selected as the lower limit.

The durations of the overshoots for each pair, level A and autocorrelation parameter β, were determined and the sample mean $\overline{X}$ and standard deviation S^2 were computed as usual. Table 1 contains the means for each pair (A,β), and Table 2, the standard deviations.

TABLE 1

OBSERVED MEANS

A \ β	.005	.010	.025	.050	.075	.100	.25	.50	.75	1.0	2.5	5.0
0.	23.138	18.824	12.159	9.668	7.737	6.651	4.347	3.286	2.892	2.606	2.073	1.948
.5	17.242	12.478	8.566	6.644	5.376	4.746	3.105	2.327	2.032	1.888	1.515	1.442
.75	14.575	11.196	7.791	5.821	4.653	4.213	2.630	2.025	1.786	1.670	1.340	1.294
1.0	12.670	11.651	6.829	5.196	4.099	3.758	2.288	1.801	1.605	1.507	1.229	1.194
1.25	13.576	9.214	6.219	4.416	3.584	3.386	2.038	1.620	1.455	1.375	1.144	1.126
1.50	11.877	8.512	6.206	4.013	3.311	3.020	1.874	1.480	1.374	1.270	1.095	1.078
1.75	10.141	8.129	5.104	3.865	3.010	2.701	1.721	1.416	1.290	1.223	1.057	1.037
2.0	10.620	7.100	4.478	3.222	2.705	2.390	1.555	1.356	1.236	1.162	1.037	1.019
2.5	5.185	7.111	2.643	3.364	2.120	1.901	1.383	1.241	1.105	1.130	1.015	1.007
3.0	3.250	7.500	2.667	4.417	1.400	1.692	1.176	1.208	1.000	1.000	1.000	1.000
3.5	0.000	1.000	4.000	2.250	0.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
4.0	0.000	0.000	0.000	4.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000

TABLE 2

OBSERVED STANDARD DEVIATIONS

A \ β	.005	.010	.025	.050	.075	.100	.25	.50	.75	1.0	2.5	5.0
0.	32.661	28.034	18.518	14.530	10.476	9.036	4.568	2.973	2.463	2.118	1.487	1.330
.5	27.634	21.336	13.252	9.466	7.030	6.179	2.978	1.894	1.519	1.366	0.881	0.783
.75	24.271	19.563	12.140	8.007	5.933	5.330	2.389	1.554	1.277	1.142	0.666	0.613
1.0	21.876	19.744	11.191	6.919	4.949	4.496	1.960	1.267	1.055	0.927	0.528	0.480
1.25	23.898	15.889	9.776	5.708	4.401	3.832	1.663	1.064	0.828	0.755	0.394	0.374
1.50	20.570	15.246	7.890	5.277	3.775	3.278	1.448	0.930	0.731	0.618	0.313	0.284
1.75	17.806	13.087	6.137	5.484	3.067	2.930	1.199	0.830	0.631	0.570	0.253	0.195
2.0	17.702	10.302	5.176	4.388	2.529	2.236	0.988	0.701	0.593	0.427	0.188	0.135
2.5	6.481	13.055	2.635	4.941	1.818	1.594	0.924	0.555	0.387	0.377	0.123	0.084
3.0	1.785	7.018	1.670	3.968	0.663	0.991	0.513	0.406	0.000	0.000	0.000	0.000
3.5	0.000	0.000	0.000	1.714	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Our next step was to plot the mean $\overline{X}$ vs. the crossing level A and the standard deviation S vs. the crossing level. Several of these curves are shown in the figures 3 and 4. Our search for estimating equations thus began. The curves indicated an exponential trend, becoming flatter and converging as the β value increased. So we graphed the same data on log paper. These plots were much straighter, but there was still some curvature left in them. Suspecting an exponential characteristic again, we tried log-log paper; the results were not satisfactory. We decided that the curvature left from the log paper could probably be taken care of with a quadratic function.

We ran polynomial regression analyses, BMD (1973), for $\overline{X}$ as a function of A and $\ln\overline{X}$ as a function of A. We concluded that we could best fit the curves using $\ln\overline{X}$ as a quadratic function of A. We did the same for the standard deviation S and concluded that our best fit would be obtained by considering S as a quadratic function of A. We thus had the following:

$$\ln\overline{X} = a_0 + a_1A + a_2A^2 \quad \text{and} \qquad \text{(III.1)}$$

$$S = b_0 + b_1A + b_2A^2,$$ where the coefficients $a_i, b_i, i=0,1,2$ were dependent on β, the autocorrelation parameter.

Table 3 gives the simple correlation coefficients for equations (III.1) for each β value. Of course, the presence of the quadratic term did, in every case, increase the correlation between observed and predicted values.

FIGURE 3

MEANS VS. CROSSING LEVELS

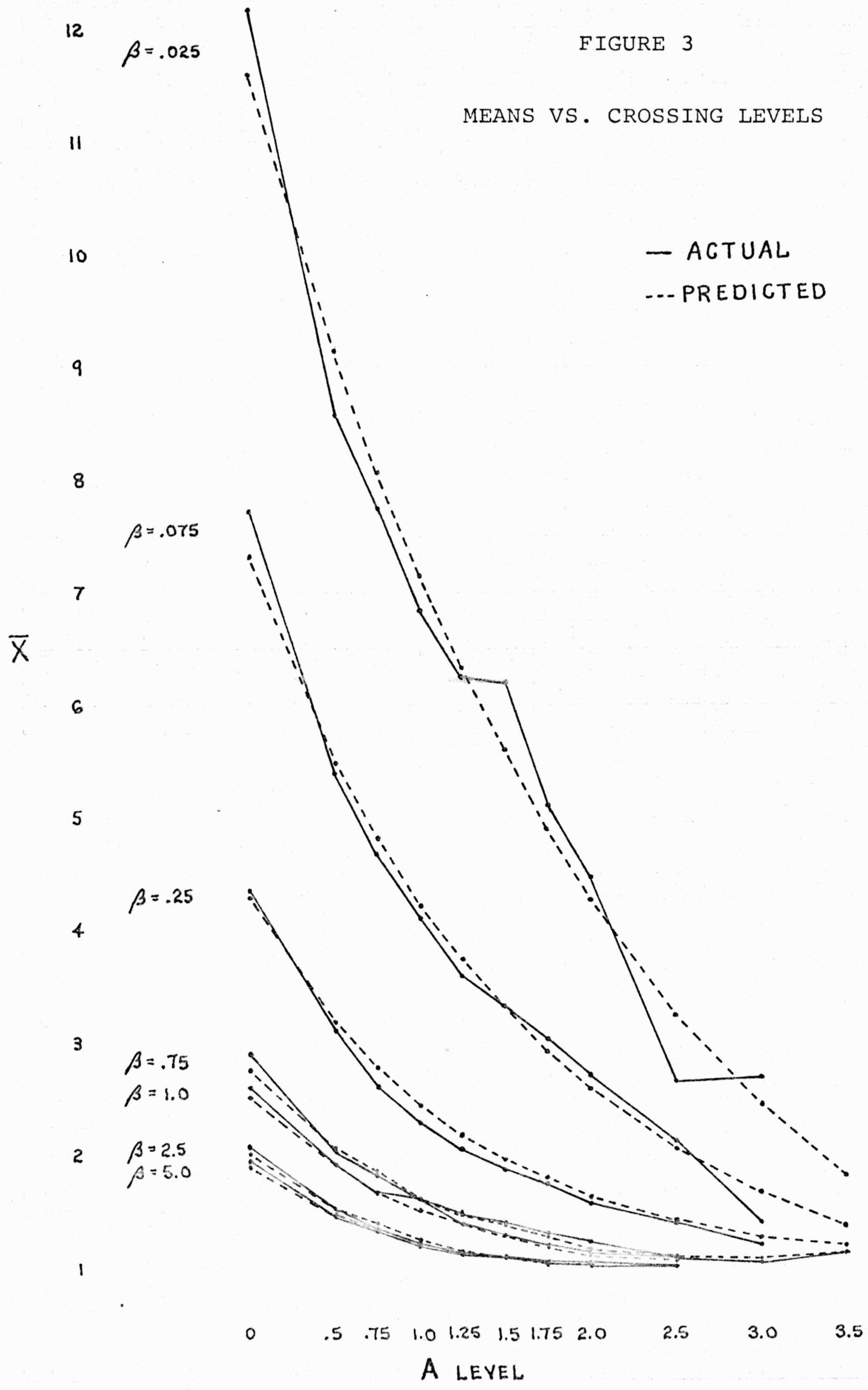


FIGURE 4

STANDARD DEVIATIONS VS. CROSSING LEVELS

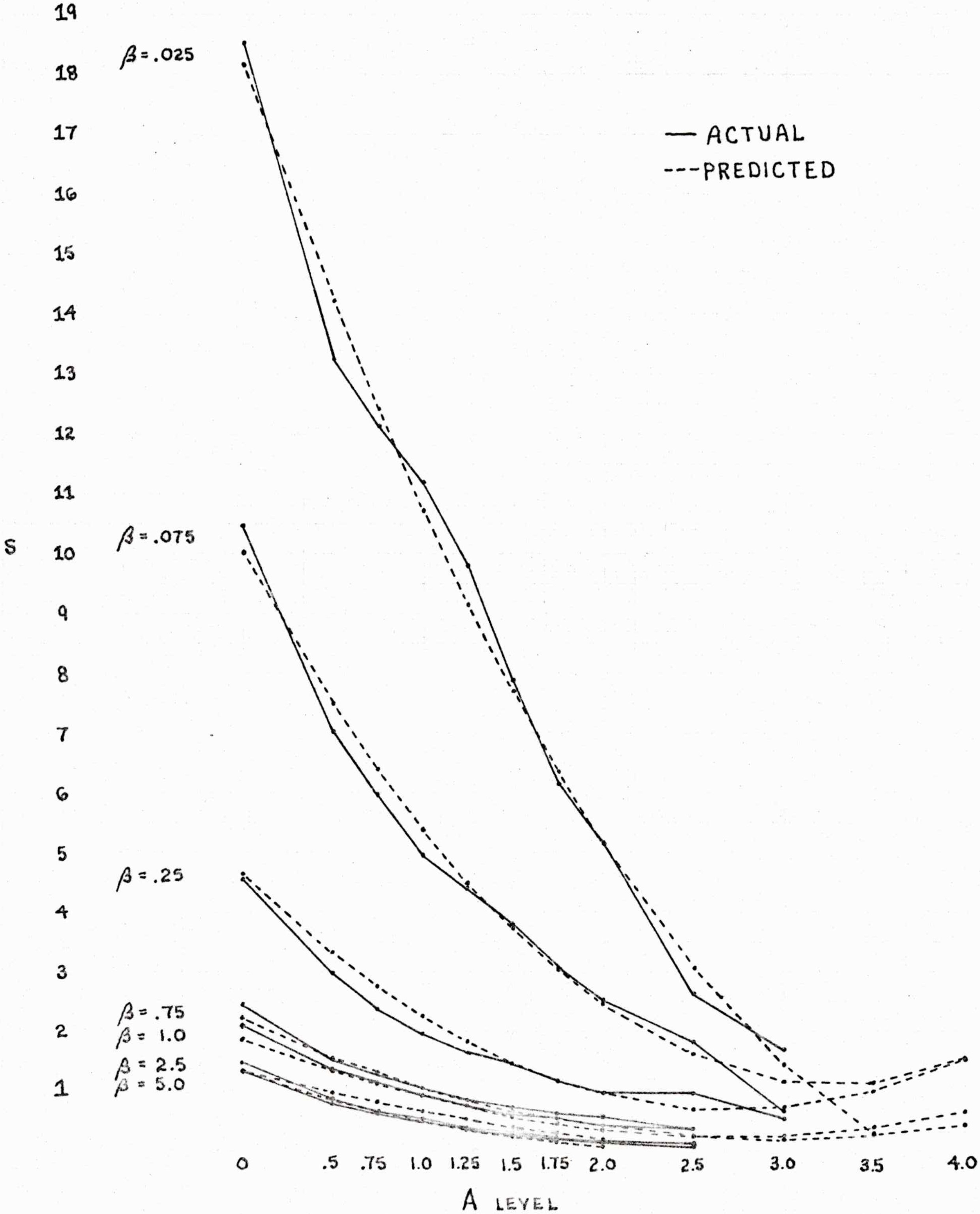


TABLE 3

SIMPLE CORRELATION COEFFICIENTS

Beta	For Means	For Standard Deviations
.005	-0.95346	-0.96940
.010	-0.90484	-0.94986
.025	-0.98143	-0.98375
.050	-0.81609	-0.85630
.075	-0.99292	-0.95819
.100	-0.99394	-0.95653
.25	-0.97928	-0.91802
.50	-0.93697	-0.91145
.75	-0.95889	-0.94079
1.0	-0.94336	-0.94624
2.5	-0.87894	-0.91315
5.0	-0.87083	-0.91963

It is clear from the table that the models (equations III.1) are adequate for each β value.

At this point, <u>for each β</u>, we had the coefficients a_0, a_1, a_2 for the mean and b_0, b_1, b_2 for the standard deviation. To determine the relationship between β and the constant terms for the mean, we plotted a_0 as a function of β. There was little doubt that this curve behaved in an exponential manner. The same phenomenon was observed in plots of other coefficients as functions of β for both the mean and standard deviation. We considered the coefficients as functions of $\ln\beta$, and then ran polynomial regression analyses of degree two and three. Choosing the third degree, we obtained very good fits. Hence we have the following:

$$a_i = c_0 + c_1(\ln\beta) + c_2(\ln\beta)^2 + c_3(\ln\beta)^3, \quad \text{and}$$
$$b_i = d_0 + d_1(\ln\beta) + d_2(\ln\beta)^2 + d_3(\ln\beta)^3, \qquad \text{(III.2)}$$

for $i = 0,1,2.$

The coefficients c_i and d_i, $i=0,1,2,3$, are presented in Table 4 for each coefficient a_i, b_i, $i=0,1,2.$

TABLE 4

Prediction Coefficients

Means	c_0	c_1	c_2	c_3
a_0	0.92590	-0.31129	0.06417	0.00988
a_1	-0.64060	0.02881	0.02002	-0.00026
a_2	0.11763	0.01749	-0.01017	-0.00131

Standard Deviations	d_0	d_1	d_2	d_3
b_0	1.88589	-1.02364	0.56796	-0.09535
b_1	-1.17242	0.74806	-0.38702	-0.01600
b_2	0.20194	-0.16723	0.07649	0.01833

Using the last set of coefficients, we were able to predict the values $a_0, a_1, a_2, b_0, b_1, b_2$ for each β value. Hence, for each β value and each A level, using (III.2) and substituting into (III.1), we could predict $\overline{X}$ and S as functions of β and A.

In summary, we have:

$\overline{X}$ predicted $= \exp[a_0(\ln\beta) + a_1(\ln\beta)\cdot A + a_2(\ln\beta)\cdot A^2]$, and

S predicted $= b_0(\ln\beta) + b_1(\ln\beta)\cdot A + b_2(\ln\beta)\cdot A^2$, where

$a_0(\ln\beta)$ is a function of $\ln\beta$, etc.

Figures 3 and 4 picture some of the observed curves together with the corresponding predicted curves for the means and standard deviations, respectively. As can be seen from the plots, for both the means and the standard deviations, the predicted curves match the observed curves quite well. For the means, the predicted values at A=0 are, consistently, slightly less than the observed values. Throughout the range of A values, the predicted curves for the means and standard deviations appear to fit equally well. Overall, the fits were much better than one would normally expect in such an analysis.

An interesting phenomenon was observed while viewing the plots of the linear and quadratic coefficient curves. Figures 5 and 6 picture the plots of the linear coefficients a_1 vs. the quadratic coefficients a_2 for the means and standard deviations, respectively. There is definitely a high correlation between these values. The author has no concrete explanation to proffer. It may be simply coincidence, it may be related to the polynomial regression program used, or it may be a common occurence in such a situation. From a statistical standpoint, the similarity means that given the linear coefficient, we are also given prior knowledge of the quadratic coefficient. Thus the linear and quadratic terms would not be totally independent. At any rate, we do not believe that our results were significantly affected by this event.

FIGURE 5

LINEAR COEFFICIENTS VS. QUADRATIC COEFFICIENTS

FOR THE MEANS

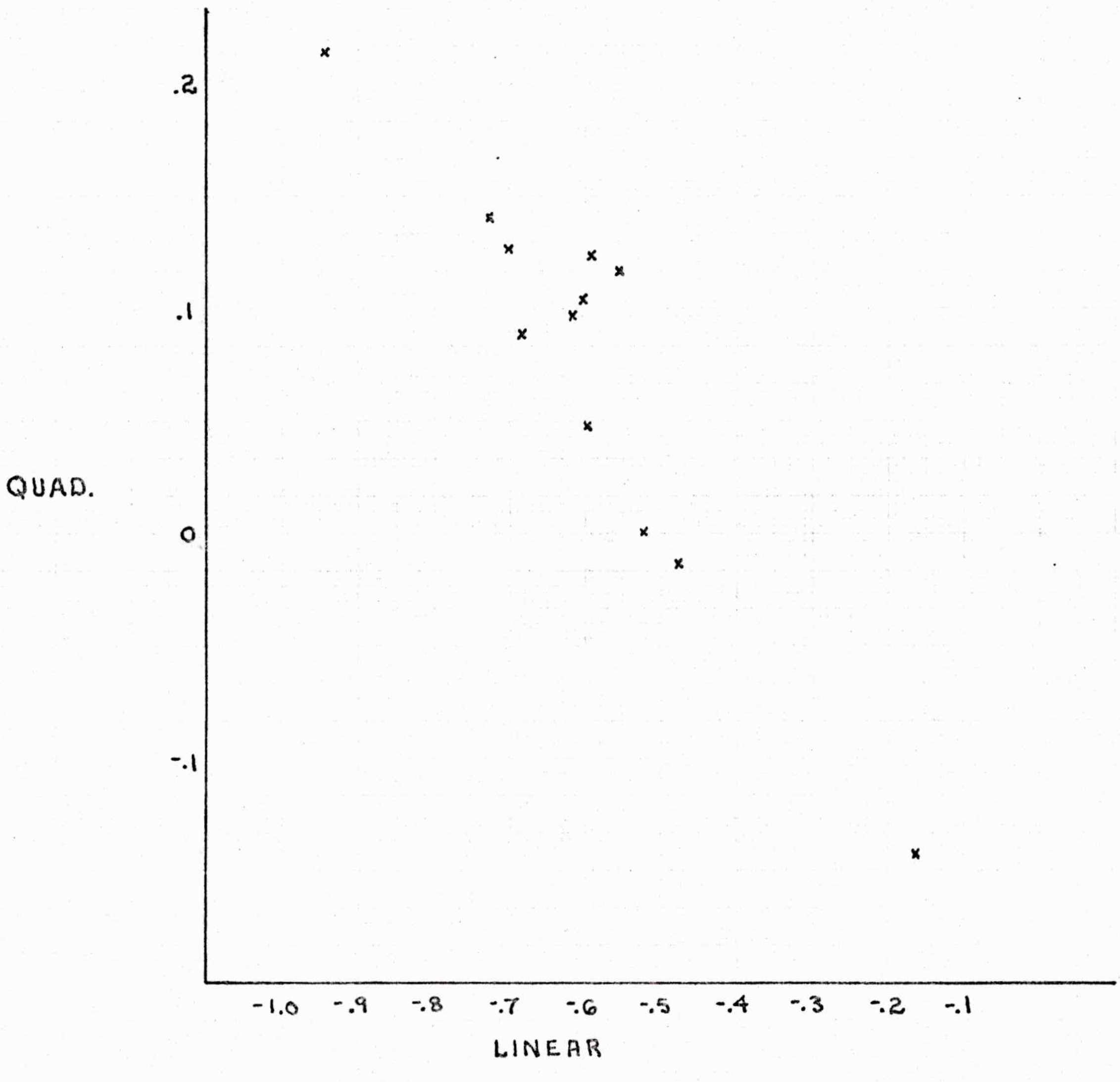
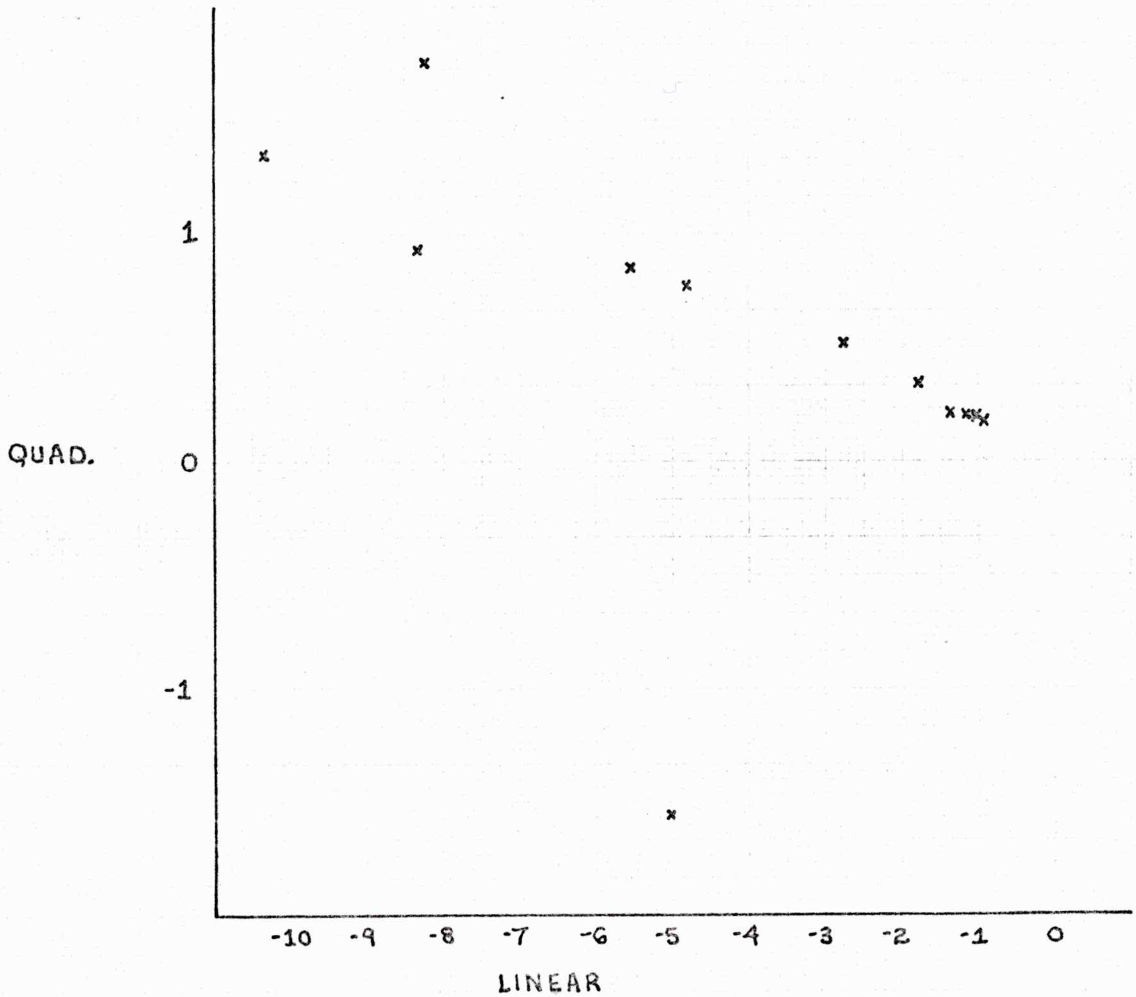


FIGURE 6

LINEAR COEFFICIENTS VS. QUADRATIC COEFFICIENTS

FOR THE STANDARD DEVIATIONS



We constructed frequency tables for each A level within each β, and then made plots of observed cumulative frequencies. Some of these are presented in Appendix A. The cumulative plots resembled some type of gamma distribution. We decided to first try the exponential distribution which is a special case of the gamma. We used as estimates of theta 1/observed mean, 1/observed standard deviation, 1/predicted mean, and 1/predicted standard deviation. The results were not altogether bad, but the estimated frequencies were consistently high for low A levels. We next tried the gamma distribution whose density function is

$$f(x;a,b) = \frac{b^a}{\Gamma(a)} \cdot x^{a-1} \cdot e^{-bx}, \text{ for } x>0, \text{ 0 elsewhere.}$$

The gamma distribution is a two parameter distribution (the exponential is a special case) and therefore provides us a richer family of curves to use. The gamma distribution, its many uses and properties, is well described in Johnson and Kotz (1970). The mean μ and variance σ^2 are given by:

$\mu = a/b$ and $\sigma^2 = a/b^2$. Noting that $\frac{\mu}{\sigma^2} = b$ and $\frac{\mu^2}{\sigma^2} = a$, when given the process mean $\overline{X}$ and variance S^2, we can, by the method of moments, easily estimate the parameters a and b for the gamma distribution pdf for any of our processes.

Using the corrected mean $(\overline{X} - 1/2)$, we utilized the gamma distribution with parameters $a = (\overline{X} -.5)^2/S^2$ and $b = (\overline{X} -.5)/S^2$ to find the predicted probabilities of overshoots of duration 1,2,3, etc. For example, for a particular β value and A level, we used either the predicted or observed values of the mean and variance to calculate a and b. Then, to determine the probability of the occurence of an overshoot of duration 1, we integrated as follows:

$$\text{Probability} = \int_{1/2}^{3/2} \frac{b^a}{\Gamma(a)} \cdot x^{a-1} \cdot e^{-bx} \cdot dx.$$

$$= \int_{0}^{1} \frac{b^a}{\Gamma(a)} \cdot (x - .5)^{a-1} \cdot e^{-b(x-.5)} dx.$$

We then used the Kolmogorov-Smirnov goodness of fit test to determine our degree of accuracy. The results were quite favorable. Appendix B presents the results of the distribution fitting for the A and β values presented in Appendix A.

Table 5 gives an overall scenario of the goodness of fit results. Upon viewing the table several points become clear. The gamma distribution gives excellent results for most A and β combinations. Especially important is the consistently excellent results for A values above 1.5--the range of A values utilized most frequently. The combination of A and β values where the gamma model fails is quite interesting. When comparing the predicted mean and variance with the observed values one notes little difference and, in fact, the gamma model using the observed values was not satisfactory either.

There are three, possibly related, reasons why the model fails in these cases. The first is the very conservative nature of the Kolmogorov-Smirnov test when the model parameters are estimated from the data (indirectly using the predicting equations) and the data are discrete although the model is a continuous one. The second reason is the possible inadequacy of the gamma model. A survey of Appendix B shows the rejection always occurs at the first frequency because the gamma model underestimates the frequency. This suggests that a more complex model (perhaps a four parameter gamma) might be more satisfactory. However, the third reason is, in the author's opinion, the real culprit. Quite simply, the sample sizes are too large. It is an interesting phenomenon in statistics that in goodness of fit problems too much data poses as many problems as does too little data. Lancaster (1969, pp. 174-75) discusses this problem and recommends utilization of moderate sample sizes. In no case where the fit is bad would it have been so if the same cumulative proportions had been observed with sample sizes under 500.

Of course, to actually verify that this phenomenon actually causes the problem would be difficult, if not impossible. However, it is pertinent to note that the difficulty starts when the sample sizes become large while the mean and variance predictions remain quite accurate and, as one can observe in Appendix A, the actual shape of the sample cumulative distribution curves remain consistent.

TABLE 5

Condensed Summary of Fitted Models

Using Predictive Equations

*Good Fit at $\alpha \leq .01$

**Good Fit at $\alpha \leq .05$

0 No data

$\mu\uparrow$ Bad Fit due to mean overestimation

$\mu\downarrow$ Bad Fit due to mean underestimation

$\sigma^2\uparrow$ Bad Fit due to variance overestimation

$\sigma^2\downarrow$ Bad Fit due to variance underestimation

β \ A	0.	.5	.75	1.0	1.25	1.5	1.75	2.0	2.5	3.0	3.5	4.0
.005	$\mu\uparrow$	**	**	*	**	**	**	**	**	0	0	0
.010	$\mu\downarrow$	*	*	**	**	*	**	**	**	0	0	0
.025	**	*	**	**	**	*	**	**	**	0	0	0
.050	**	*	**	**	**	**	**	**	**	**	**	0
.075	**	*	**	**	**	**	**	**	**	**	0	0
.10	**	**	**	**	**	**	**	**	**	**	0	0
.25	**	**	**	*	$\sigma^2\uparrow$	*	*	*	**	**	0	0
.50	*	*	$\sigma^2\uparrow$	$\sigma^2\uparrow$	$\sigma^2\uparrow$	*	*	**	**	**	0	0
.75	*	$\sigma^2\uparrow$	$\sigma^2\uparrow$	$\sigma^2\uparrow$	$\sigma^2\uparrow$	$\sigma^2\uparrow$	**	**	**	**	**	0
1.0	$\sigma^2\downarrow$	$\sigma^2\downarrow$	$\sigma^2\downarrow$	$\sigma^2\downarrow$	$\sigma^2\downarrow$	$\sigma^2\downarrow$	**	**	**	**	**	**
2.5	$\sigma^2\downarrow$	$\sigma^2\downarrow$	$\sigma^2\downarrow$	$\sigma^2\downarrow$	$\sigma^2\downarrow$	$\sigma^2\uparrow$	**	**	**	**	**	0
5.0	**	$\sigma^2\uparrow$	$\sigma^2\uparrow$	$\sigma^2\uparrow$	**	**	**	**	**	**	0	0

CHAPTER 4

Applications

This chapter presents the procedures to utilize the results obtained in Chapter 3. The computer program developed to do the necessary calculations is presented in Appendix C.

The analysis was conducted using a standard normal process ($\mu = 0$, $\sigma^2 = 1$) and duration times above a level A were counted. However, a simple transformation permits the use of any mean and variance, and duration times below a specified level can be addressed using the symmetry of the normal distribution.

A final note involves the time intervals. In most processes the sampling and estimation procedures use data gathered at regular intervals and the results are, to some extent, dependent on the interval widths. Questions must be posed in this context, e.g., it is not reasonable to request the probability of a duration time exceeding five seconds when the autocorrelation parameter is estimated based on samples at one minute intervals--the autocorrelation parameter estimate must also be based on (or modified to correspond to) data gathered at one second intervals.

Our simulation process, hence the β values, used unit intervals corresponding to serial correlation coefficients of lag 1, lag 2, etc. Should a β value using the actual time intervals be given, it must be modified to represent the autocorrelation parameter of a process sampled at unit intervals and the time period of these intervals must agree with the time period referred to in the problem under consideration.

This transformation procedure is summarized as follows. Assume a value β' has been calculated using intervals $\tau' = 0, L, 2L, 3L \ldots$ giving $R(\tau') = \exp(-\beta'|\tau|)$. This corresponds to the function $R(\tau) = \exp(-\beta' h|\tau|)$ where $\tau = 0,1,2,\ldots$. Thus $\beta = \beta' h$ would be the autocorrelation parameter estimate corresponding to intervals of unit length.

As an example, assume a wind speed process where $\mu = 24$ m/s, $\sigma = 8$ m/s, and $\beta' = .026$ based on samples measured at 10 minute intervals. We desire to calculate the probability of exceeding 30 m/s for 3.2 minutes. The β' value is for $\tau' = 0$, 10 sec., 20 sec.,... . The quantity $\beta = \beta' h = .026 * 10 = .26$ is for $\tau = 0$,1 sec., 2 sec.,... and is the value we would utilize in calculating the probability.

The computer program required the following inputs:

1) Process mean μ,
2) Process standard deviation σ,
3) Process autocorrelation parameter (modified to reflect intervals of unit length if necessary),
4) The crossing level L under consideration, and
5) The number of time units x (perhaps fractional) that the duration lasts.

The output gives

1) The μ, σ, β, and L values,
2) The adjusted crossing level A $(= |L-\mu|/\sigma)$,
3) Predicted mean and variance of the process, and
4) Pr {Exceeding L for x time units}.

Referring to the example in the preceding paragraph the inputs would be:

1) $\mu = 24$,

2) $\sigma = 8$,

3) $\beta = .26$, (the .026 value was modified),

4) $L = 30$, and

5) $x = 3.2$.

Table 6 gives the computer results corresponding to these inputs.

The question concerning the applicability of these results to nonnormal processes remains. The answer must necessarily be the same as given in Carter and Madison (1973, pp. 35-36).

TABLE 6

PREDICTION PROGRAM OUTPUT

```
AUTOCORRELATION PARAMETER =      0.260

CROSSING LEVEL     =     30.000   ADJUSTED LEVEL           =      0.750

PROCESS MEAN       =     24.000   DISTRIBUTION MEAN        =      2.718

PROCESS VARIANCE   =     64.000   DISTRIBUTION VARIANCE    =      7.232

        PROBABILITY OF EXCEEDING THE CROSSING LEVEL FOR
                  3.200 TIME UNITS = 0.309
```

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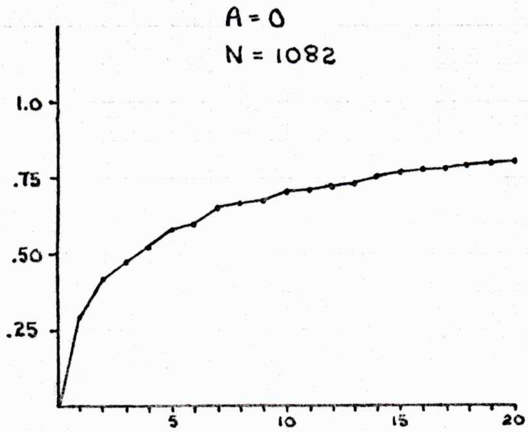
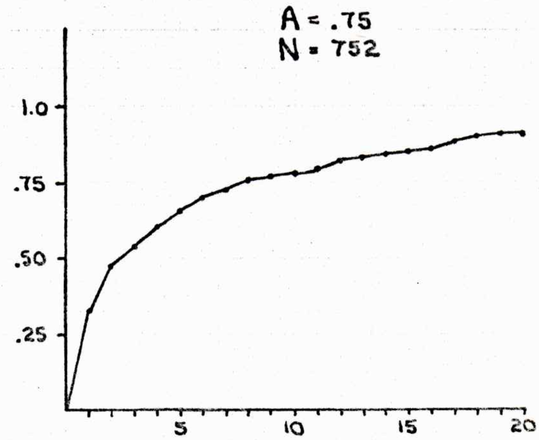
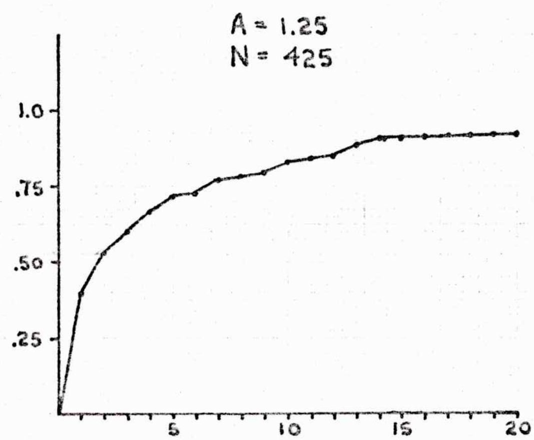
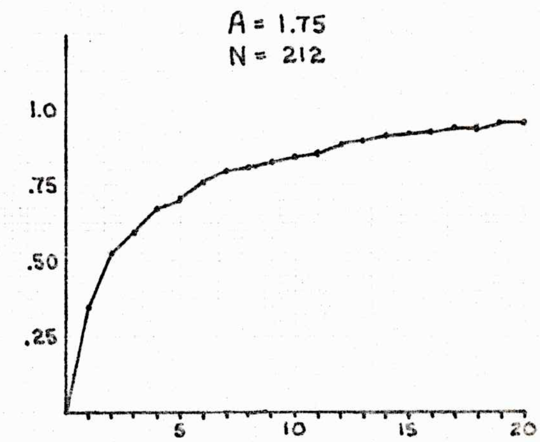
APPENDIX A

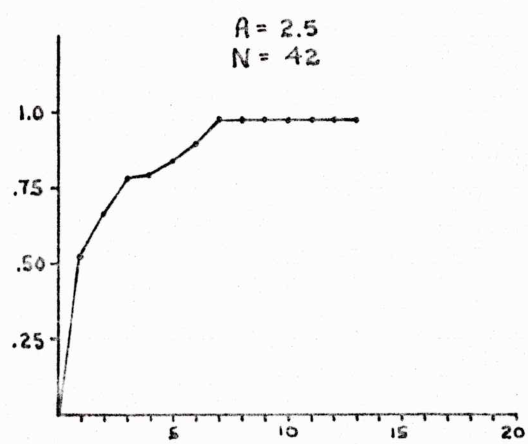
SAMPLE CUMULATIVE DISTRIBUTION CURVES FOR AUTOCORRELATION PARAMETERS: .025, .075, .25, .75, 1.0, 2.5, 5.0 AND A-LEVELS: 0., .75, 1.25, 1.75, 2.5


A = 0
N = 1082
1.0
.75
.50
.25
5
10
15
20

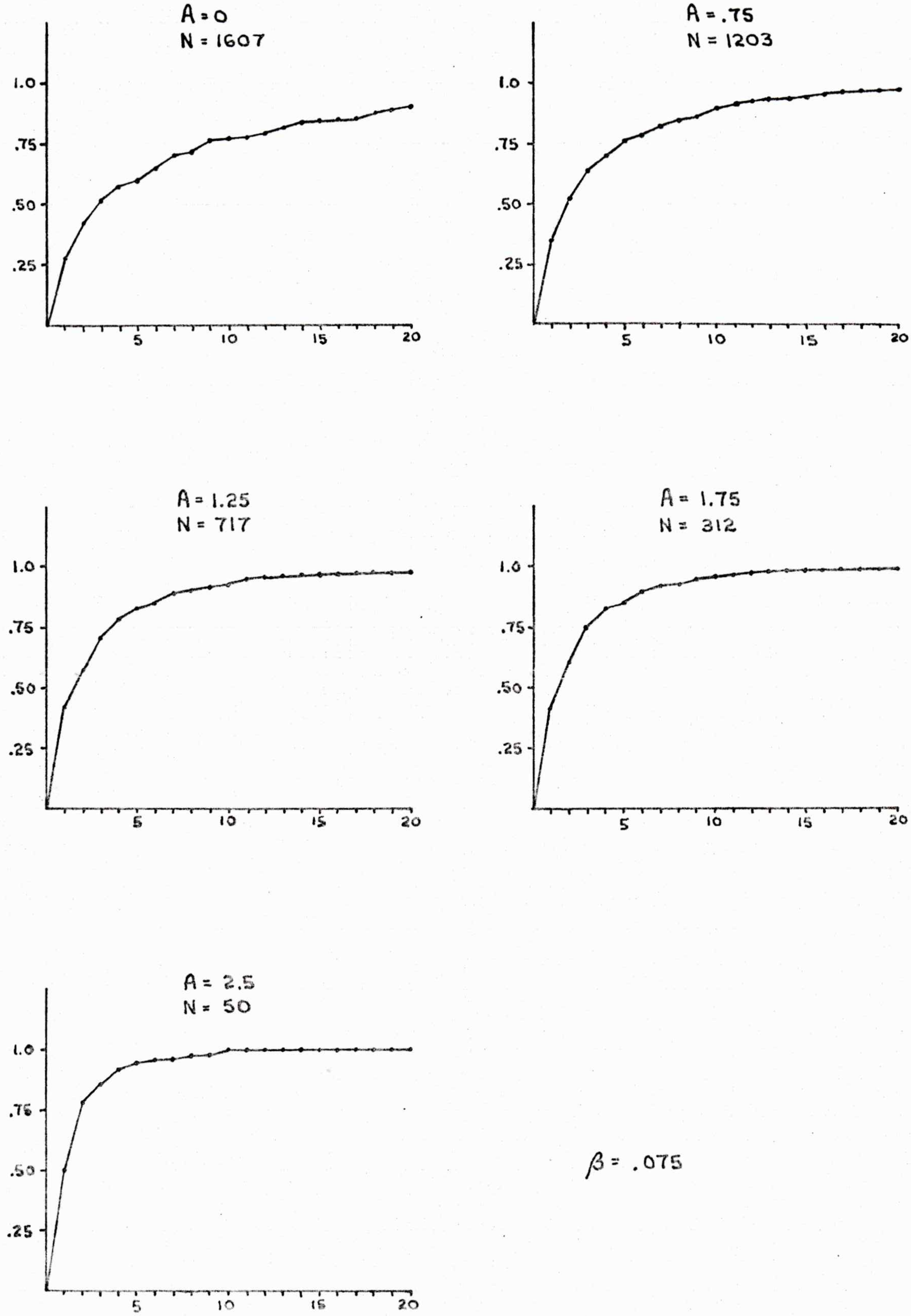

A = .75
N = 752
1.0
.75
.50
.25
5
10
15
20

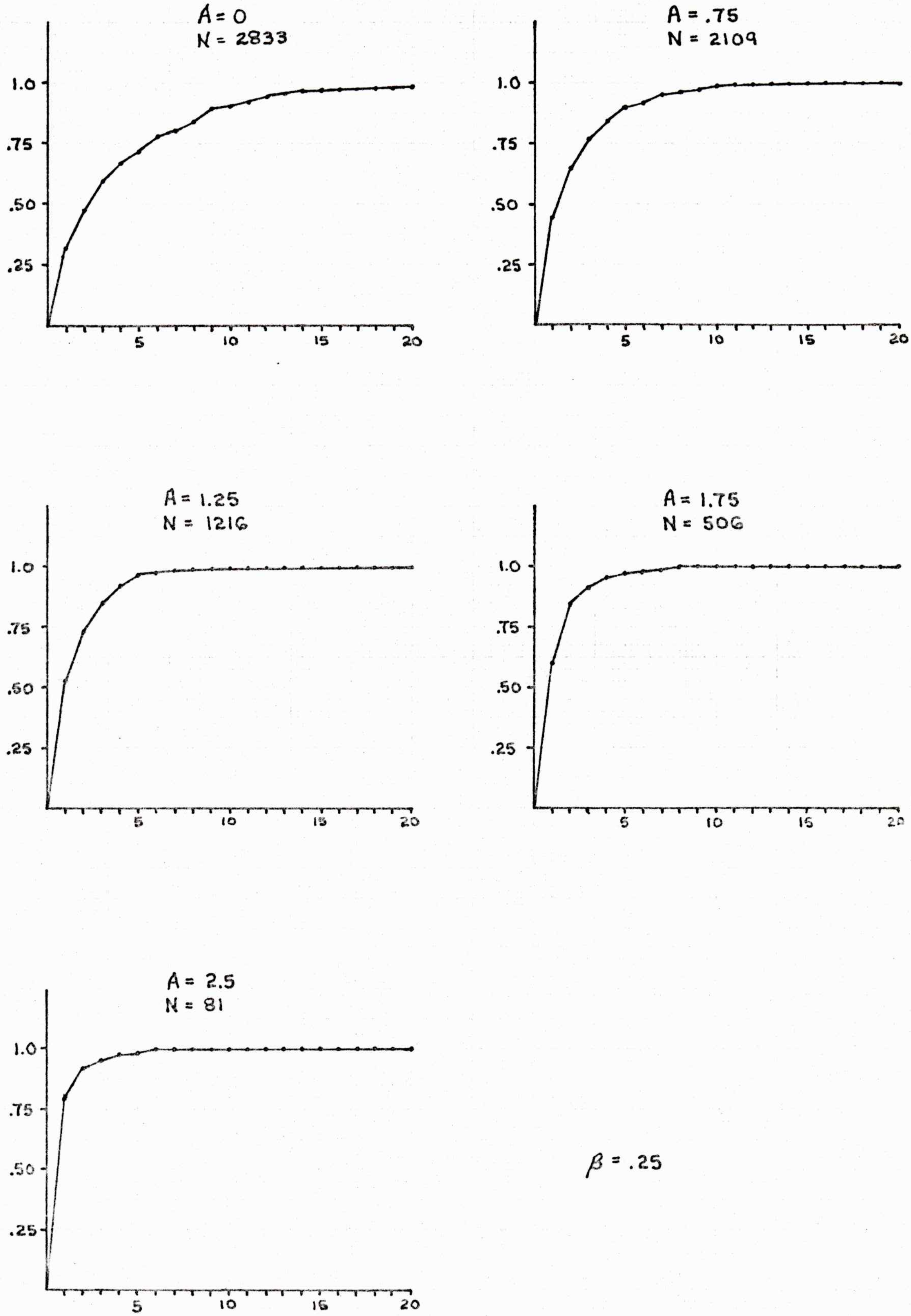

A = 1.25
N = 425
1.0
.75
.50
.25
5
10
15
20

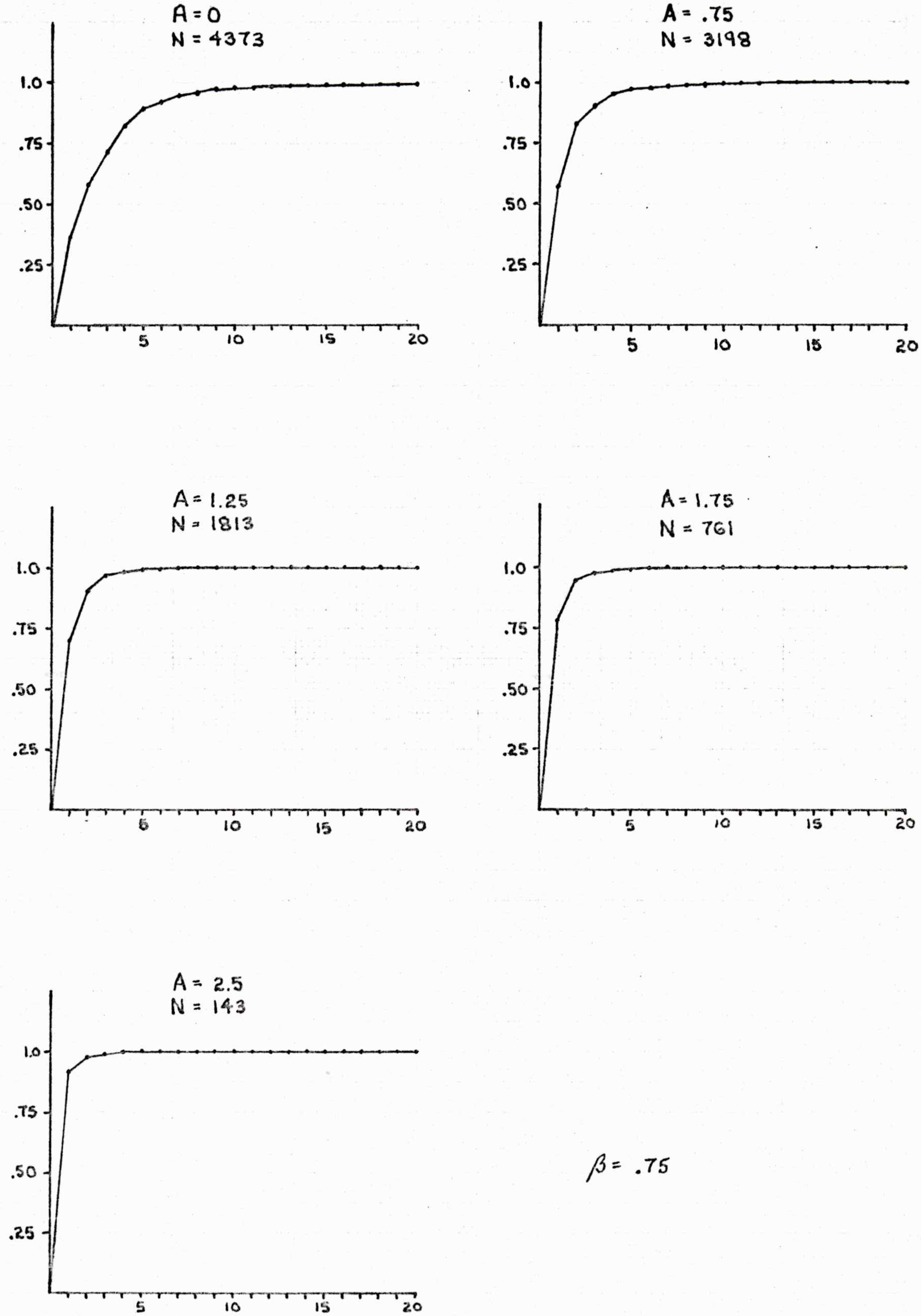

A = 1.75
N = 212
1.0
.75
.50
.25
5
10
15
20

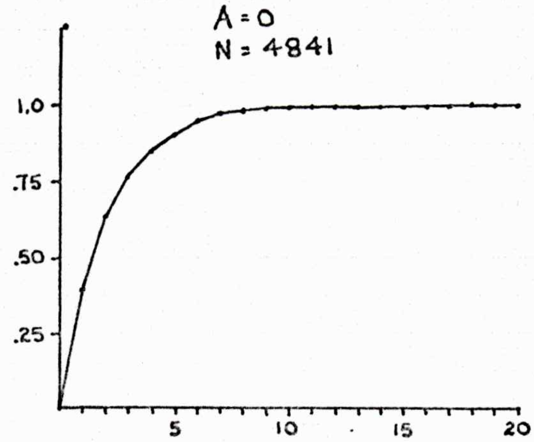

A = 2.5
N = 42
1.0
.75
.50
.25
5
10
15
20

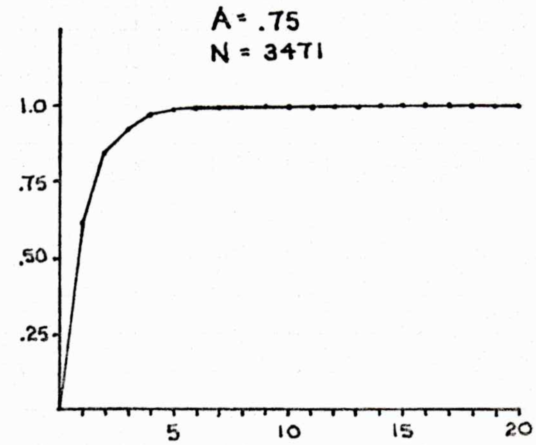
β = .025

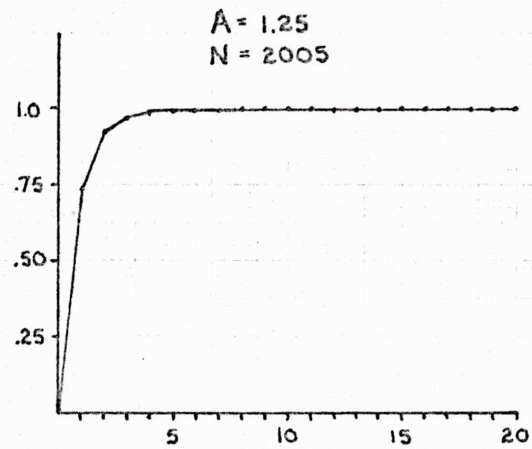

A = 0
N = 1607
A = .75
N = 1203
A = 1.25
N = 717
A = 1.75
N = 312
A = 2.5
N = 50
1.0
.75
.50
.25
5
10
15
20
β = .075

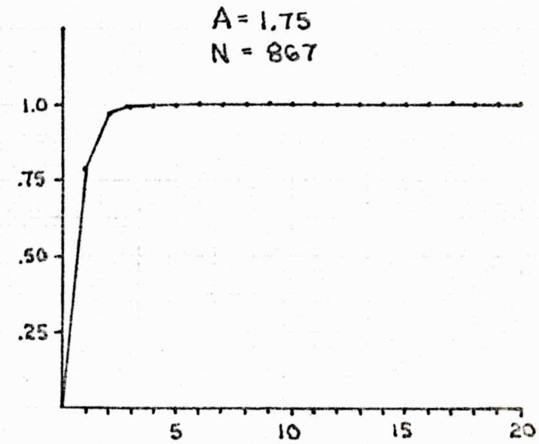

A = 0
N = 2833
A = .75
N = 2109
A = 1.25
N = 1216
A = 1.75
N = 506
A = 2.5
N = 81
1.0
.75
.50
.25
5
10
15
20
β = .25

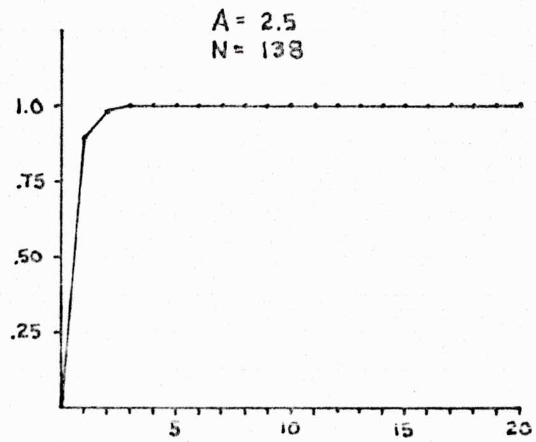

A = 0
N = 4373
A = .75
N = 3198
A = 1.25
N = 1813
A = 1.75
N = 761
A = 2.5
N = 143
1.0
.75
.50
.25
5
10
15
20
β = .75


A = 0
N = 4841
1.0
.75
.50
.25
5
10
15
20

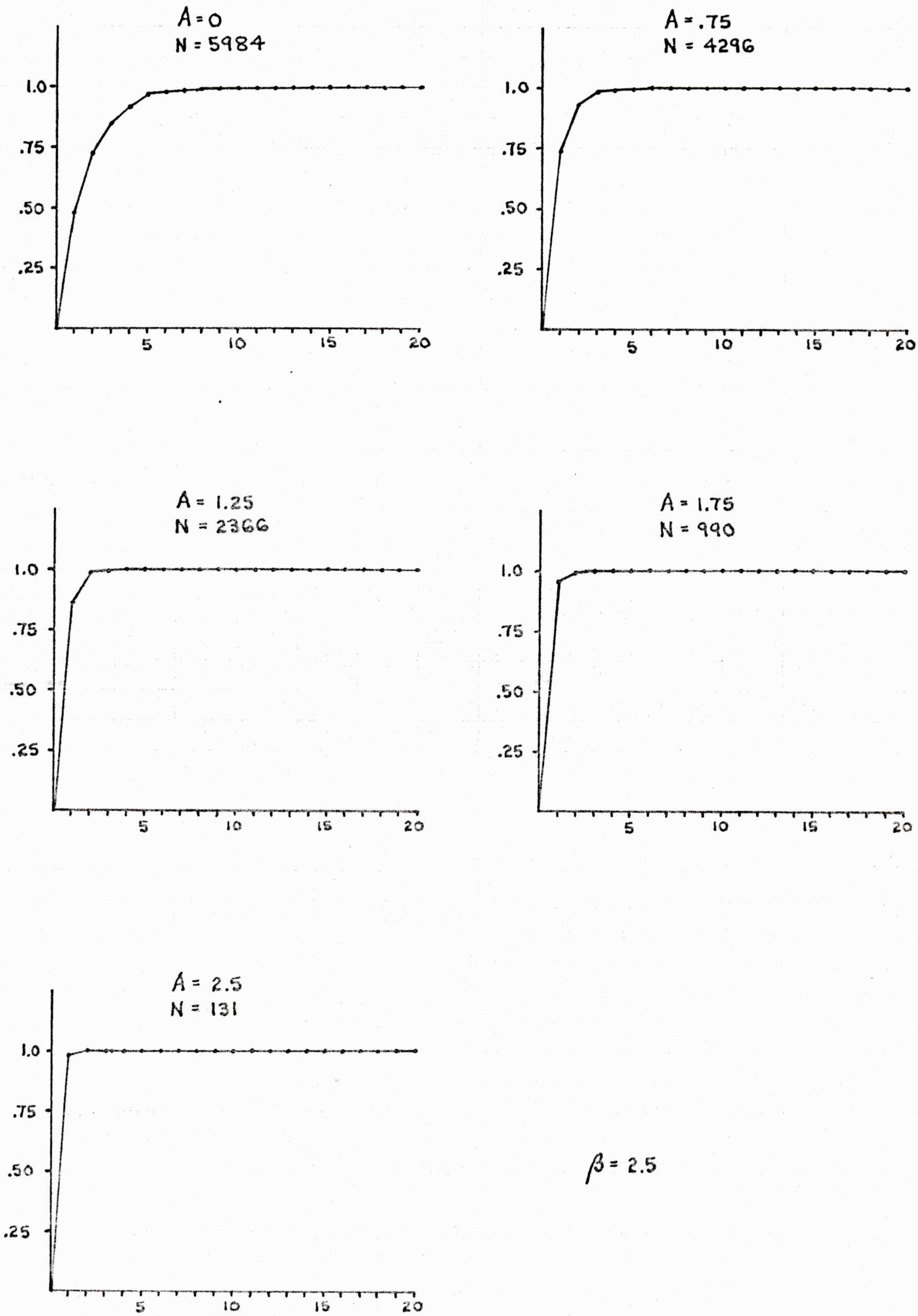

A = .75
N = 3471
1.0
.75
.50
.25
5
10
15
20

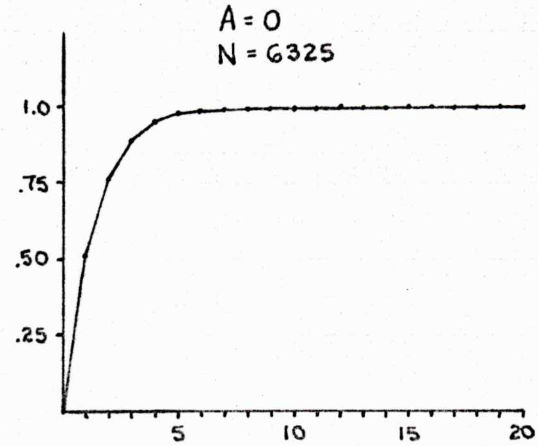

A = 1.25
N = 2005
1.0
.75
.50
.25
5
10
15
20

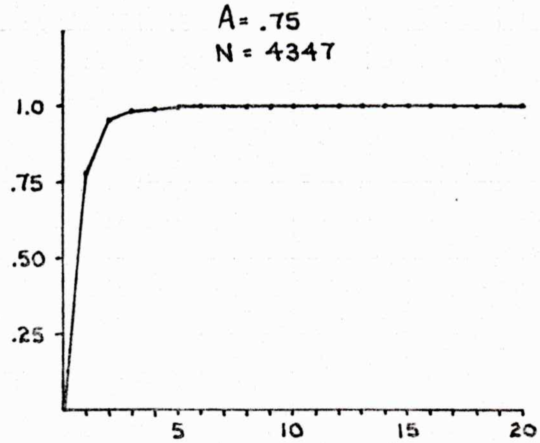

A = 1.75
N = 867
1.0
.75
.50
.25
5
10
15
20

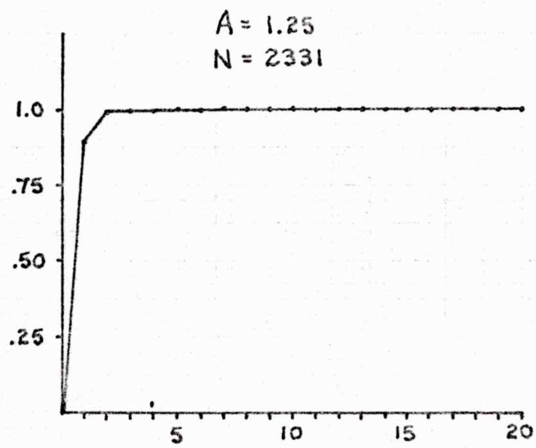

A = 2.5
N = 138
1.0
.75
.50
.25
5
10
15
20

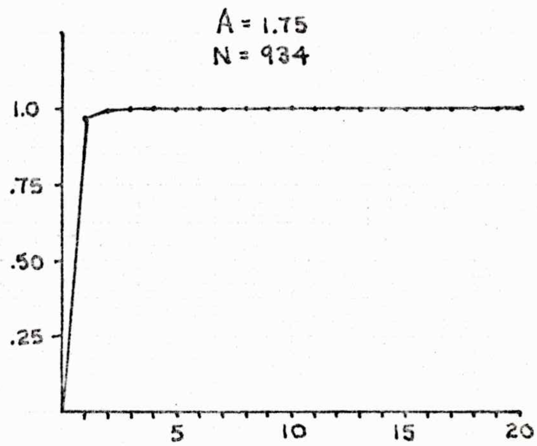
$\beta = 1.0$

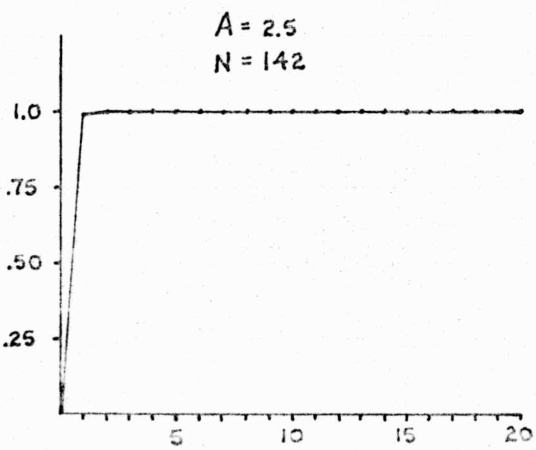

A = 0
N = 5984
A = .75
N = 4296
A = 1.25
N = 2366
A = 1.75
N = 990
A = 2.5
N = 131
1.0
.75
.50
.25
5
10
15
20
β = 2.5


A = 0
N = 6325
1.0
.75
.50
.25
5
10
15
20


A = .75
N = 4347
1.0
.75
.50
.25
5
10
15
20


A = 1.25
N = 2331
1.0
.75
.50
.25
5
10
15
20


A = 1.75
N = 934
1.0
.75
.50
.25
5
10
15
20


A = 2.5
N = 142
1.0
.75
.50
.25
5
10
15
20

$\beta = 5.0$

APPENDIX B

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TESTS

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 0.000   AUTOCORRELATION PARAMETER = 0.025

OBSERVED MEAN = 12.15896   OBSERVED VARIANCE = 342.92211
PREDICTED MEAN = 11.60541   PREDICTED VARIANCE = 330.40380

GAMMA DIST PARAMETERS: ALPHA = 0.37327   BETA = 0.03361

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	306	0.28281	0.31413	-0.03132
2	140	0.41220	0.40324	0.00896
3	72	0.47874	0.46498	0.01377
4	54	0.52865	0.51314	0.01551
5	46	0.57116	0.55287	0.01829
6	36	0.60444	0.58672	0.01771
7	42	0.64325	0.61619	0.02707
8	28	0.66913	0.64223	0.02690
9	15	0.68299	0.66550	0.01749
10	19	0.70055	0.68649	0.01407
11	11	0.71072	0.70554	0.00518
12	23	0.73198	0.72295	0.00903
13	12	0.74307	0.73892	0.00415
14	9	0.75139	0.75364	-0.00225
15	14	0.76433	0.76724	-0.00292
16	8	0.77172	0.77986	-0.00814
17	12	0.78281	0.79159	-0.00878
18	10	0.79205	0.80252	-0.01047
19	15	0.80591	0.81273	-0.00682
20	10	0.81516	0.82228	-0.00712
OVER 20	200	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 1082

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 0.750   AUTOCORRELATION PARAMETER = 0.025

OBSERVED MEAN = 7.79122   OBSERVED VARIANCE = 147.37265
PREDICTED MEAN = 8.12091   PREDICTED VARIANCE = 153.98528

GAMMA DIST PARAMETERS: ALPHA = 0.37717   BETA = 0.04949

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	239	0.31782	0.35727	-0.03945
2	114	0.46941	0.45791	0.01150
3	62	0.55186	0.52666	0.02520
4	44	0.61037	0.57952	0.03085
5	37	0.65957	0.62248	0.03710
6	29	0.69814	0.65853	0.03961
7	23	0.72872	0.68943	0.03929
8	18	0.75266	0.71632	0.03633
9	9	0.76463	0.74000	0.02463
10	14	0.78324	0.76101	0.02223
11	14	0.80186	0.77981	0.02205
12	10	0.81516	0.79670	0.01845
13	12	0.83112	0.81197	0.01914
14	12	0.84707	0.82582	0.02125
15	10	0.86037	0.83843	0.02194
16	8	0.87101	0.84994	0.02107
17	6	0.87899	0.86047	0.01852
18	3	0.88298	0.87014	0.01284
19	4	0.88830	0.87902	0.00928
20	3	0.89229	0.88720	0.00508
OVER 20	81	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 752

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 1.250 AUTOCORRELATION PARAMETER = 0.025

OBSERVED MEAN = 6.21882 OBSERVED VARIANCE = 95.57799
PREDICTED MEAN = 6.32318 PREDICTED VARIANCE = 83.74115

GAMMA DIST PARAMETERS: ALPHA = 0.40493 BETA = 0.06954

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	164	0.38588	0.37552	0.01036
2	61	0.52941	0.48763	0.04178
3	38	0.61882	0.56381	0.05501
4	22	0.67059	0.62177	0.04882
5	17	0.71059	0.66825	0.04234
6	15	0.74588	0.70670	0.03918
7	9	0.76706	0.73916	0.02790
8	10	0.79059	0.76696	0.02363
9	5	0.80235	0.79102	0.01133
10	13	0.83294	0.81203	0.02091
11	6	0.84706	0.83049	0.01657
12	7	0.86353	0.84680	0.01673
13	7	0.88000	0.86128	0.01872
14	4	0.88941	0.87418	0.01523
15	4	0.89882	0.88571	0.01311
16	0	0.89882	0.89605	0.00277
17	2	0.90353	0.90534	-0.00181
18	2	0.90824	0.91371	-0.00547
19	2	0.91294	0.92126	-0.00832
20	2	0.91765	0.92809	-0.01044
OVER 20	35	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 425

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 1.750 AUTOCORRELATION PARAMETER = 0.025

OBSERVED MEAN = 5.10377 OBSERVED VARIANCE = 37.66846
PREDICTED MEAN = 4.87560 PREDICTED VARIANCE = 40.48451

GAMMA DIST PARAMETERS: ALPHA = 0.47292 BETA = 0.10808

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	76	0.35849	0.38100	-0.02251
2	35	0.52358	0.51154	0.01205
3	19	0.61321	0.60003	0.01318
4	12	0.66981	0.66632	0.00349
5	7	0.70283	0.71838	-0.01555
6	11	0.75472	0.76037	-0.00566
7	9	0.79717	0.79488	0.00229
8	2	0.80660	0.82359	-0.01698
9	3	0.82075	0.84771	-0.02695
10	1	0.82547	0.86812	-0.04265
11	6	0.85377	0.88550	-0.03172
12	7	0.88679	0.90036	-0.01357
13	3	0.90094	0.91313	-0.01219
14	2	0.91038	0.92414	-0.01376
15	1	0.91509	0.93365	-0.01855
16	3	0.92925	0.94189	-0.01264
17	2	0.93868	0.94905	-0.01037
18	0	0.93868	0.95527	-0.01659
19	3	0.95283	0.96070	-0.00787
20	0	0.95283	0.96544	-0.01261
OVER 20	10	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 212

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 2.500 AUTOCORRELATION PARAMETER = 0.025

OBSERVED MEAN = 2.64286 OBSERVED VARIANCE = 6.94388
PREDICTED MEAN = 3.24129 PREDICTED VARIANCE = 9.37086

GAMMA DIST PARAMETERS: ALPHA = 0.80192 BETA = 0.29253

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	22	0.52381	0.35289	0.17092
2	6	0.66667	0.54617	0.12049
3	5	0.78571	0.67599	0.10972
4	1	0.80952	0.76651	0.04301
5	2	0.85714	0.83075	0.02639
6	1	0.88095	0.87681	0.00414
7	4	0.97619	0.91007	0.06612
8	0	0.97619	0.93419	0.04200
9	0	0.97619	0.95175	0.02444
10	0	0.97619	0.96457	0.01162
11	0	0.97619	0.97395	0.00224
12	0	0.97619	0.98083	-0.00464
13	0	0.97619	0.98587	-0.00968
14	1	1.00000	0.98958	0.01042
15	0	1.00000	0.99231	0.00769
16	0	1.00000	0.99433	0.00567
17	0	1.00000	0.99581	0.00419
18	0	1.00000	0.99690	0.00310
19	0	1.00000	0.99771	0.00229
20	0	1.00000	0.99830	0.00170
OVER 20	0	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 42

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 0.000   AUTOCORRELATION PARAMETER = 0.075

OBSERVED MEAN = 7.73678   OBSERVED VARIANCE = 109.75586
PREDICTED MEAN = 7.32331   PREDICTED VARIANCE = 100.10458

GAMMA DIST PARAMETERS: ALPHA = 0.46509   BETA = 0.06816

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	463	0.28811	0.31691	-0.02880
2	231	0.43186	0.42836	0.00350
3	126	0.51027	0.50670	0.00357
4	101	0.57312	0.56763	0.00549
5	70	0.61668	0.61732	-0.00064
6	67	0.65837	0.65899	-0.00062
7	62	0.69695	0.69456	0.00239
8	52	0.72931	0.72534	0.00397
9	42	0.75544	0.75222	0.00322
10	29	0.77349	0.77588	-0.00239
11	24	0.78843	0.79682	-0.00840
12	27	0.80523	0.81546	-0.01023
13	20	0.81767	0.83210	-0.01443
14	21	0.83074	0.84702	-0.01628
15	12	0.83821	0.86044	-0.02223
16	19	0.85003	0.87253	-0.02250
17	23	0.86434	0.88345	-0.01911
18	18	0.87554	0.89334	-0.01779
19	9	0.88114	0.90230	-0.02115
20	25	0.89670	0.91044	-0.01374
OVER 20	166	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 1607

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 0.750   AUTOCORRELATION PARAMETER = 0.075

OBSERVED MEAN = 4.65253   OBSERVED VARIANCE = 35.19763
PREDICTED MEAN = 4.82534   PREDICTED VARIANCE = 40.96538

GAMMA DIST PARAMETERS: ALPHA = 0.45669   BETA = 0.10559

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	430	0.35744	0.39145	-0.03401
2	200	0.52369	0.52048	0.00321
3	127	0.62926	0.60740	0.02186
4	80	0.69576	0.67233	0.02343
5	67	0.75145	0.72322	0.02823
6	54	0.79634	0.76425	0.03209
7	33	0.82377	0.79795	0.02582
8	33	0.85121	0.82600	0.02521
9	25	0.87199	0.84957	0.02242
10	18	0.88695	0.86953	0.01742
11	19	0.90274	0.88654	0.01620
12	9	0.91022	0.90111	0.00912
13	14	0.92186	0.91363	0.00823
14	7	0.92768	0.92444	0.00324
15	9	0.93516	0.93379	0.00137
16	10	0.94347	0.94191	0.00156
17	6	0.94846	0.94897	-0.00051
18	10	0.95677	0.95512	0.00165
19	5	0.96093	0.96049	0.00044
20	7	0.96675	0.96519	0.00156
OVER 20	40	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 1203

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 1.250   AUTOCORRELATION PARAMETER = 0.075

OBSERVED MEAN = 3.58438   OBSERVED VARIANCE = 19.36978
PREDICTED MEAN = 3.71563   PREDICTED VARIANCE = 20.39255

GAMMA DIST PARAMETERS: ALPHA = 0.50706   BETA = 0.15769

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	304	0.42399	0.41976	0.00423
2	123	0.59554	0.56756	0.02798
3	79	0.70572	0.66461	0.04111
4	53	0.77964	0.73461	0.04503
5	25	0.81450	0.78736	0.02715
6	30	0.85635	0.82813	0.02821
7	22	0.88703	0.86019	0.02684
8	12	0.90377	0.88570	0.01806
9	13	0.92190	0.90619	0.01571
10	6	0.93026	0.92274	0.00752
11	8	0.94142	0.93620	0.00522
12	6	0.94979	0.94719	0.00260
13	1	0.95119	0.95620	-0.00501
14	8	0.96234	0.96361	-0.00126
15	4	0.96792	0.96971	-0.00179
16	3	0.97211	0.97476	-0.00266
17	2	0.97490	0.97894	-0.00405
18	3	0.97908	0.98241	-0.00333
19	1	0.98047	0.98529	-0.00482
20	0	0.98047	0.98769	-0.00722
OVER 20	14	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 717

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 1.750   AUTOCORRELATION PARAMETER = 0.075

OBSERVED MEAN = 3.00961   OBSERVED VARIANCE = 9.40695
PREDICTED MEAN = 2.89981   PREDICTED VARIANCE = 9.27860

GAMMA DIST PARAMETERS: ALPHA = 0.62068   BETA = 0.25864

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	129	0.41346	0.43796	-0.02450
2	65	0.62179	0.61528	0.00651
3	40	0.75000	0.72709	0.02291
4	22	0.82051	0.80286	0.01765
5	13	0.86218	0.85598	0.00620
6	8	0.88782	0.89396	-0.00614
7	8	0.91346	0.92147	-0.00801
8	4	0.92628	0.94159	-0.01531
9	5	0.94231	0.95640	-0.01409
10	5	0.95833	0.96735	-0.00902
11	2	0.96474	0.97550	-0.01076
12	5	0.98077	0.98157	-0.00080
13	2	0.98718	0.98612	0.00106
14	1	0.99038	0.98952	0.00086
15	1	0.99359	0.99209	0.00150
16	0	0.99359	0.99401	-0.00042
17	1	0.99679	0.99547	0.00133
18	0	0.99679	0.99656	0.00023
19	0	0.99679	0.99739	-0.00060
20	0	0.99679	0.99802	-0.00122
OVER 20	1	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 312

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 2.500 AUTOCORRELATION PARAMETER = 0.075

OBSERVED MEAN = 2.12000 OBSERVED VARIANCE = 3.30559
PREDICTED MEAN = 2.05026 PREDICTED VARIANCE = 2.62239

GAMMA DIST PARAMETERS: ALPHA = 0.91646 BETA = 0.59116

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	25	0.50000	0.48842	0.01158
2	14	0.78000	0.72594	0.05406
3	4	0.86000	0.85167	0.00833
4	3	0.92000	0.91930	0.00070
5	1	0.94000	0.95596	-0.01596
6	1	0.96000	0.97591	-0.01591
7	0	0.96000	0.98680	-0.02680
8	1	0.98000	0.99276	-0.01276
9	0	0.98000	0.99603	-0.01603
10	1	1.00000	0.99782	0.00218
11	0	1.00000	0.99880	0.00120
12	0	1.00000	0.99934	0.00066
13	0	1.00000	0.99964	0.00036
14	0	1.00000	0.99980	0.00020
15	0	1.00000	0.99989	0.00011
16	0	1.00000	0.99994	0.00006
17	0	1.00000	0.99997	0.00003
18	0	1.00000	0.99998	0.00002
19	0	1.00000	0.99999	0.00001
20	0	1.00000	0.99999	0.00001
OVER 20	0	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 50

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 0.000   AUTOCORRELATION PARAMETER = 0.250

OBSERVED MEAN = 4.34698   OBSERVED VARIANCE = 20.87042
PREDICTED MEAN = 4.28208   PREDICTED VARIANCE = 21.62711

GAMMA DIST PARAMETERS: ALPHA = 0.66140   BETA = 0.17488

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	890	0.31415	0.32685	-0.01270
2	485	0.48535	0.48415	0.00120
3	311	0.59513	0.59445	0.00068
4	218	0.67208	0.67692	-0.00484
5	187	0.73809	0.74044	-0.00235
6	135	0.78574	0.79024	-0.00451
7	130	0.83163	0.82975	0.00188
8	79	0.85951	0.86134	-0.00182
9	74	0.88563	0.88675	-0.00112
10	58	0.90611	0.90730	-0.00119
11	52	0.92446	0.92397	0.00049
12	44	0.93999	0.93755	0.00245
13	27	0.94952	0.94863	0.00090
14	26	0.95870	0.95769	0.00101
15	26	0.96788	0.96511	0.00277
16	17	0.97388	0.97121	0.00267
17	9	0.97706	0.97622	0.00084
18	13	0.98164	0.98034	0.00131
19	7	0.98412	0.98373	0.00038
20	10	0.98765	0.98654	0.00111
OVER 20	35	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 2833

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 0.750   AUTOCORRELATION PARAMETER = 0.250

OBSERVED MEAN = 2.63016   OBSERVED VARIANCE = 5.70580
PREDICTED MEAN = 2.76463   PREDICTED VARIANCE = 7.65495

GAMMA DIST PARAMETERS: ALPHA = 0.66997   BETA = 0.29584

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	929	0.44049	0.43646	0.00403
2	423	0.64106	0.62383	0.01724
3	266	0.76719	0.74067	0.02652
4	164	0.84495	0.81827	0.02668
5	110	0.89711	0.87134	0.02577
6	68	0.92935	0.90827	0.02108
7	48	0.95211	0.93426	0.01785
8	35	0.96871	0.95269	0.01601
9	23	0.97961	0.96585	0.01376
10	18	0.98815	0.97528	0.01287
11	7	0.99147	0.98207	0.00940
12	6	0.99431	0.98697	0.00734
13	1	0.99478	0.99051	0.00427
14	3	0.99621	0.99309	0.00312
15	1	0.99668	0.99495	0.00173
16	1	0.99716	0.99631	0.00084
17	3	0.99858	0.99730	0.00127
18	0	0.99858	0.99803	0.00055
19	1	0.99905	0.99855	0.00050
20	0	0.99905	0.99894	0.00011
OVER 20	2	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 2109

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 1.250   AUTOCORRELATION PARAMETER = 0.250

OBSERVED MEAN = 2.03783   OBSERVED VARIANCE = 2.76502
PREDICTED MEAN = 2.16745   PREDICTED VARIANCE = 3.39810

GAMMA DIST PARAMETERS: ALPHA = 0.81822   BETA = 0.49070

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	655	0.53865	0.48331	0.05534
2	254	0.74753	0.70469	0.04285
3	139	0.86184	0.82761	0.03424
4	73	0.92188	0.89828	0.02359
5	42	0.95641	0.93958	0.01683
6	22	0.97451	0.96395	0.01055
7	17	0.98849	0.97842	0.01007
8	3	0.99095	0.98705	0.00390
9	5	0.99507	0.99221	0.00285
10	2	0.99671	0.99531	0.00140
11	1	0.99753	0.99717	0.00036
12	2	0.99918	0.99829	0.00089
13	0	0.99918	0.99897	0.00021
14	0	0.99918	0.99938	-0.00020
15	0	0.99918	0.99962	-0.00044
16	0	0.99918	0.99977	-0.00059
17	0	0.99918	0.99986	-0.00068
18	0	0.99918	0.99992	-0.00074
19	0	0.99918	0.99995	-0.00077
20	0	0.99918	0.99997	-0.00079
OVER 20	1	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 1216

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 1.750   AUTOCORRELATION PARAMETER = 0.250

OBSERVED MEAN = 1.72134   OBSERVED VARIANCE = 1.43816
PREDICTED MEAN = 1.76625   PREDICTED VARIANCE = 1.40658

GAMMA DIST PARAMETERS: ALPHA = 1.13992   BETA = 0.90023

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	307	0.60672	0.53131	0.07541
2	115	0.83399	0.79747	0.03653
3	41	0.91502	0.91418	0.00084
4	22	0.95850	0.96400	-0.00550
5	10	0.97826	0.98498	-0.00672
6	5	0.98814	0.99376	-0.00562
7	5	0.99802	0.99742	0.00061
8	1	1.00000	0.99893	0.00107
9	0	1.00000	0.99956	0.00044
10	0	1.00000	0.99982	0.00018
11	0	1.00000	0.99993	0.00007
12	0	1.00000	0.99997	0.00003
13	0	1.00000	0.99999	0.00001
14	0	1.00000	0.99999	0.00001
15	0	1.00000	1.00000	0.00000
16	0	1.00000	1.00000	0.00000
17	0	1.00000	1.00000	0.00000
18	0	1.00000	1.00000	0.00000
19	0	1.00000	1.00000	0.00000
20	0	1.00000	1.00000	0.00000
OVER 20	0	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 506

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 2.500   AUTOCORRELATION PARAMETER = 0.250

OBSERVED MEAN = 1.38272   OBSERVED VARIANCE = 0.85353
PREDICTED MEAN = 1.39699   PREDICTED VARIANCE = 0.48802

GAMMA DIST PARAMETERS: ALPHA = 1.64867   BETA = 1.83802

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	64	0.79012	0.65623	0.13389
2	10	0.91358	0.92382	-0.01024
3	3	0.95062	0.98494	-0.03432
4	2	0.97531	0.99718	-0.02187
5	1	0.98765	0.99949	-0.01184
6	1	1.00000	0.99991	0.00009
7	0	1.00000	0.99998	0.00002
8	0	1.00000	1.00000	0.00000
9	0	1.00000	1.00000	0.00000
10	0	1.00000	1.00000	0.00000
11	0	1.00000	1.00000	0.00000
12	0	1.00000	1.00000	0.00000
13	0	1.00000	1.00000	0.00000
14	0	1.00000	1.00000	0.00000
15	0	1.00000	1.00000	0.00000
16	0	1.00000	1.00000	0.00000
17	0	1.00000	1.00000	0.00000
18	0	1.00000	1.00000	0.00000
19	0	1.00000	1.00000	0.00000
20	0	1.00000	1.00000	0.00000
OVER 20	0	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 81

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 0.000   AUTOCORRELATION PARAMETER = 0.750

OBSERVED MEAN = 2.89229   OBSERVED VARIANCE = 6.06546
PREDICTED MEAN = 2.77466   PREDICTED VARIANCE = 4.97132

GAMMA DIST PARAMETERS: ALPHA = 1.04078   BETA = 0.45756

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	1628	0.37228	0.34775	0.02453
2	912	0.58084	0.58128	-0.00045
3	610	0.72033	0.73232	-0.01199
4	412	0.81454	0.82925	-0.01470
5	280	0.87857	0.89123	-0.01265
6	161	0.91539	0.93078	-0.01539
7	122	0.94329	0.95598	-0.01269
8	93	0.96456	0.97202	-0.00746
9	43	0.97439	0.98222	-0.00783
10	39	0.98331	0.98871	-0.00540
11	20	0.98788	0.99283	-0.00495
12	14	0.99108	0.99545	-0.00437
13	12	0.99383	0.99711	-0.00329
14	10	0.99611	0.99817	-0.00206
15	8	0.99794	0.99884	-0.00090
16	2	0.99840	0.99926	-0.00086
17	2	0.99886	0.99953	-0.00068
18	1	0.99909	0.99970	-0.00062
19	0	0.99909	0.99981	-0.00073
20	1	0.99931	0.99988	-0.00057
OVER 20	3	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 4373

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 0.750   AUTOCORRELATION PARAMETER = 0.750

OBSERVED MEAN = 1.78643   OBSERVED VARIANCE = 1.62950
PREDICTED MEAN = 1.81846   PREDICTED VARIANCE = 1.71390

GAMMA DIST PARAMETERS: ALPHA = 1.01425   BETA = 0.76927

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	1860	0.58161	0.52991	0.05171
2	743	0.81395	0.78086	0.03308
3	305	0.90932	0.89804	0.01127
4	141	0.95341	0.95261	0.00080
5	75	0.97686	0.97799	-0.00113
6	39	0.98906	0.98978	-0.00072
7	20	0.99531	0.99525	0.00006
8	7	0.99750	0.99780	-0.00030
9	3	0.99844	0.99898	-0.00054
10	2	0.99906	0.99953	-0.00046
11	1	0.99937	0.99978	-0.00041
12	1	0.99969	0.99990	-0.00021
13	1	1.00000	0.99995	0.00005
14	0	1.00000	0.99998	0.00002
15	0	1.00000	0.99999	0.00001
16	0	1.00000	1.00000	0.00000
17	0	1.00000	1.00000	0.00000
18	0	1.00000	1.00000	0.00000
19	0	1.00000	1.00000	0.00000
20	0	1.00000	1.00000	0.00000
OVER 20	0	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 3198

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 1.250 AUTOCORRELATION PARAMETER = 0.750

OBSERVED MEAN = 1.45505 OBSERVED VARIANCE = 0.68482
PREDICTED MEAN = 1.47133 PREDICTED VARIANCE = 0.73183

GAMMA DIST PARAMETERS: ALPHA = 1.28920 BETA = 1.32726

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	1254	0.69167	0.62912	0.06256
2	383	0.90292	0.88694	0.01598
3	116	0.96691	0.96711	-0.00021
4	38	0.98787	0.99065	-0.00278
5	16	0.99669	0.99738	-0.00069
6	4	0.99890	0.99927	-0.00037
7	2	1.00000	0.99980	0.00020
8	0	1.00000	0.99994	0.00006
9	0	1.00000	0.99998	0.00002
10	0	1.00000	1.00000	0.00000
11	0	1.00000	1.00000	0.00000
12	0	1.00000	1.00000	0.00000
13	0	1.00000	1.00000	0.00000
14	0	1.00000	1.00000	0.00000
15	0	1.00000	1.00000	0.00000
16	0	1.00000	1.00000	0.00000
17	0	1.00000	1.00000	0.00000
18	0	1.00000	1.00000	0.00000
19	0	1.00000	1.00000	0.00000
20	0	1.00000	1.00000	0.00000
OVER 20	0	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 1813

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 1.750   AUTOCORRELATION PARAMETER = 0.750

OBSERVED MEAN = 1.29041   OBSERVED VARIANCE = 0.39792
PREDICTED MEAN = 1.25889   PREDICTED VARIANCE = 0.28063

GAMMA DIST PARAMETERS: ALPHA = 2.05222   BETA = 2.70423

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	595	0.78187	0.74012	0.04175
2	125	0.94612	0.96907	-0.02295
3	30	0.98555	0.99700	-0.01146
4	9	0.99737	0.99974	-0.00236
5	1	0.99869	0.99998	-0.00129
6	1	1.00000	1.00000	0.00000
7	0	1.00000	1.00000	0.00000
8	0	1.00000	1.00000	0.00000
9	0	1.00000	1.00000	0.00000
10	0	1.00000	1.00000	0.00000
11	0	1.00000	1.00000	0.00000
12	0	1.00000	1.00000	0.00000
13	0	1.00000	1.00000	0.00000
14	0	1.00000	1.00000	0.00000
15	0	1.00000	1.00000	0.00000
16	0	1.00000	1.00000	0.00000
17	0	1.00000	1.00000	0.00000
18	0	1.00000	1.00000	0.00000
19	0	1.00000	1.00000	0.00000
20	0	1.00000	1.00000	0.00000
OVER 20	0	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 761

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 2.500 AUTOCORRELATION PARAMETER = 0.750

OBSERVED MEAN = 1.10489 OBSERVED VARIANCE = 0.14984
PREDICTED MEAN = 1.10643 PREDICTED VARIANCE = 0.07902

GAMMA DIST PARAMETERS: ALPHA = 4.65376 BETA = 7.67402

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	131	0.91608	0.90746	0.00862
2	10	0.98601	0.99959	-0.01357
3	1	0.99301	1.00000	-0.00699
4	1	1.00000	1.00000	0.00000
5	0	1.00000	1.00000	0.00000
6	0	1.00000	1.00000	0.00000
7	0	1.00000	1.00000	0.00000
8	0	1.00000	1.00000	0.00000
9	0	1.00000	1.00000	0.00000
10	0	1.00000	1.00000	0.00000
11	0	1.00000	1.00000	0.00000
12	0	1.00000	1.00000	0.00000
13	0	1.00000	1.00000	0.00000
14	0	1.00000	1.00000	0.00000
15	0	1.00000	1.00000	0.00000
16	0	1.00000	1.00000	0.00000
17	0	1.00000	1.00000	0.00000
18	0	1.00000	1.00000	0.00000
19	0	1.00000	1.00000	0.00000
20	0	1.00000	1.00000	0.00000
OVER 20	0	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 143

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 0.000   AUTOCORRELATION PARAMETER = 1.000

OBSERVED MEAN = 2.60607   OBSERVED VARIANCE = 4.48415
PREDICTED MEAN = 2.52414   PREDICTED VARIANCE = 3.55658

GAMMA DIST PARAMETERS: ALPHA = 1.15199   BETA = 0.56912

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	1925	0.39765	0.36326	0.03438
2	1104	0.62570	0.61857	0.00713
3	689	0.76802	0.77530	-0.00728
4	412	0.85313	0.86879	-0.01567
5	262	0.90725	0.92381	-0.01656
6	178	0.94402	0.95592	-0.01190
7	101	0.96488	0.97457	-0.00969
8	59	0.97707	0.98536	-0.00829
9	43	0.98595	0.99159	-0.00564
10	18	0.98967	0.99517	-0.00550
11	18	0.99339	0.99723	-0.00384
12	9	0.99525	0.99842	-0.00317
13	5	0.99628	0.99909	-0.00281
14	7	0.99773	0.99948	-0.00175
15	5	0.99876	0.99970	-0.00094
16	3	0.99938	0.99983	-0.00045
17	2	0.99979	0.99990	-0.00011
18	1	1.00000	0.99994	0.00006
19	0	1.00000	0.99997	0.00003
20	0	1.00000	0.99998	0.00002
OVER 20	0	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 4841

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 0.750 AUTOCORRELATION PARAMETER = 1.000

OBSERVED MEAN = 1.66955 OBSERVED VARIANCE = 1.30336
PREDICTED MEAN = 1.66799 PREDICTED VARIANCE = 1.25477

GAMMA DIST PARAMETERS: ALPHA = 1.08720 BETA = 0.93084

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	2151	0.61971	0.56719	0.05252
2	770	0.84154	0.82263	0.01891
3	302	0.92855	0.92822	0.00033
4	140	0.96889	0.97114	-0.00225
5	61	0.98646	0.98844	-0.00198
6	21	0.99251	0.99538	-0.00287
7	15	0.99683	0.99816	-0.00133
8	7	0.99885	0.99927	-0.00042
9	1	0.99914	0.99971	-0.00057
10	1	0.99942	0.99988	-0.00046
11	0	0.99942	0.99995	-0.00053
12	0	0.99942	0.99998	-0.00056
13	1	0.99971	0.99999	-0.00028
14	1	1.00000	1.00000	0.00000
15	0	1.00000	1.00000	0.00000
16	0	1.00000	1.00000	0.00000
17	0	1.00000	1.00000	0.00000
18	0	1.00000	1.00000	0.00000
19	0	1.00000	1.00000	0.00000
20	0	1.00000	1.00000	0.00000
OVER 20	0	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 3471

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 1.250 AUTOCORRELATION PARAMETER = 1.000

OBSERVED MEAN = 1.37456 OBSERVED VARIANCE = 0.57043
PREDICTED MEAN = 1.36199 PREDICTED VARIANCE = 0.54155

GAMMA DIST PARAMETERS: ALPHA = 1.37205 BETA = 1.59172

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	1485	0.74065	0.67827	0.06238
2	363	0.92170	0.92107	0.00063
3	105	0.97406	0.98184	-0.00777
4	37	0.99252	0.99595	-0.00343
5	10	0.99751	0.99911	-0.00161
6	4	0.99950	0.99981	-0.00031
7	0	0.99950	0.99996	-0.00046
8	1	1.00000	0.99999	0.00001
9	0	1.00000	1.00000	0.00000
10	0	1.00000	1.00000	0.00000
11	0	1.00000	1.00000	0.00000
12	0	1.00000	1.00000	0.00000
13	0	1.00000	1.00000	0.00000
14	0	1.00000	1.00000	0.00000
15	0	1.00000	1.00000	0.00000
16	0	1.00000	1.00000	0.00000
17	0	1.00000	1.00000	0.00000
18	0	1.00000	1.00000	0.00000
19	0	1.00000	1.00000	0.00000
20	0	1.00000	1.00000	0.00000
OVER 20	0	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 2005

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 1.750   AUTOCORRELATION PARAMETER = 1.000

OBSERVED MEAN = 1.22261   OBSERVED VARIANCE = 0.32530
PREDICTED MEAN = 1.17950   PREDICTED VARIANCE = 0.20484

GAMMA DIST PARAMETERS: ALPHA = 2.25402   BETA = 3.31716

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	722	0.83276	0.79859	0.03417
2	110	0.95963	0.98515	-0.02552
3	26	0.98962	0.99915	-0.00954
4	6	0.99654	0.99996	-0.00342
5	2	0.99885	1.00000	-0.00115
6	1	1.00000	1.00000	0.00000
7	0	1.00000	1.00000	0.00000
8	0	1.00000	1.00000	0.00000
9	0	1.00000	1.00000	0.00000
10	0	1.00000	1.00000	0.00000
11	0	1.00000	1.00000	0.00000
12	0	1.00000	1.00000	0.00000
13	0	1.00000	1.00000	0.00000
14	0	1.00000	1.00000	0.00000
15	0	1.00000	1.00000	0.00000
16	0	1.00000	1.00000	0.00000
17	0	1.00000	1.00000	0.00000
18	0	1.00000	1.00000	0.00000
19	0	1.00000	1.00000	0.00000
20	0	1.00000	1.00000	0.00000
OVER 20	0	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 867

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 2.500   AUTOCORRELATION PARAMETER = 1.000

OBSERVED MEAN = 1.13043   OBSERVED VARIANCE = 0.14241
PREDICTED MEAN = 1.06140   PREDICTED VARIANCE = 0.04707

GAMMA DIST PARAMETERS: ALPHA = 6.69515   BETA = 11.92585

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	122	0.88406	0.96205	-0.07799
2	14	0.98551	0.99999	-0.01448
3	2	1.00000	1.00000	0.00000
4	0	1.00000	1.00000	0.00000
5	0	1.00000	1.00000	0.00000
6	0	1.00000	1.00000	0.00000
7	0	1.00000	1.00000	0.00000
8	0	1.00000	1.00000	0.00000
9	0	1.00000	1.00000	0.00000
10	0	1.00000	1.00000	0.00000
11	0	1.00000	1.00000	0.00000
12	0	1.00000	1.00000	0.00000
13	0	1.00000	1.00000	0.00000
14	0	1.00000	1.00000	0.00000
15	0	1.00000	1.00000	0.00000
16	0	1.00000	1.00000	0.00000
17	0	1.00000	1.00000	0.00000
18	0	1.00000	1.00000	0.00000
19	0	1.00000	1.00000	0.00000
20	0	1.00000	1.00000	0.00000
OVER 20	0	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 138

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 0.000   AUTOCORRELATION PARAMETER = 2.500

OBSERVED MEAN = 2.07269   OBSERVED VARIANCE = 2.21012
PREDICTED MEAN = 2.01808   PREDICTED VARIANCE = 1.82638

GAMMA DIST PARAMETERS: ALPHA = 1.26181   BETA = 0.83119

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	2890	0.48295	0.44830	0.03465
2	1526	0.73797	0.73182	0.00614
3	703	0.85545	0.87403	-0.01858
4	419	0.92547	0.94186	-0.01639
5	226	0.96324	0.97345	-0.01022
6	102	0.98028	0.98797	-0.00769
7	60	0.99031	0.99457	-0.00427
8	34	0.99599	0.99756	-0.00157
9	9	0.99749	0.99891	-0.00142
10	8	0.99883	0.99951	-0.00068
11	4	0.99950	0.99978	-0.00028
12	2	0.99983	0.99990	-0.00007
13	1	1.00000	0.99996	0.00004
14	0	1.00000	0.99998	0.00002
15	0	1.00000	0.99999	0.00001
16	0	1.00000	1.00000	0.00000
17	0	1.00000	1.00000	0.00000
18	0	1.00000	1.00000	0.00000
19	0	1.00000	1.00000	0.00000
20	0	1.00000	1.00000	0.00000
OVER 20	0	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 5984

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 0.750 AUTOCORRELATION PARAMETER = 2.500

OBSERVED MEAN = 1.33962 OBSERVED VARIANCE = 0.44402
PREDICTED MEAN = 1.38232 PREDICTED VARIANCE = 0.64758

GAMMA DIST PARAMETERS: ALPHA = 1.20215 BETA = 1.36249

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	3204	0.74581	0.67181	0.07400
2	813	0.93506	0.90726	0.02780
3	206	0.98301	0.97466	0.00835
4	60	0.99697	0.99319	0.00379
5	11	0.99953	0.99819	0.00135
6	2	1.00000	0.99952	0.00048
7	0	1.00000	0.99987	0.00013
8	0	1.00000	0.99997	0.00003
9	0	1.00000	0.99999	0.00001
10	0	1.00000	1.00000	0.00000
11	0	1.00000	1.00000	0.00000
12	0	1.00000	1.00000	0.00000
13	0	1.00000	1.00000	0.00000
14	0	1.00000	1.00000	0.00000
15	0	1.00000	1.00000	0.00000
16	0	1.00000	1.00000	0.00000
17	0	1.00000	1.00000	0.00000
18	0	1.00000	1.00000	0.00000
19	0	1.00000	1.00000	0.00000
20	0	1.00000	1.00000	0.00000
OVER 20	0	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 4296

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 1.250   AUTOCORRELATION PARAMETER = 2.500

OBSERVED MEAN = 1.14412   OBSERVED VARIANCE = 0.15547
PREDICTED MEAN = 1.16076   PREDICTED VARIANCE = 0.27002

GAMMA DIST PARAMETERS: ALPHA = 1.61690   BETA = 2.44703

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	2061	0.87109	0.79438	0.07671
2	271	0.98563	0.97509	0.01054
3	32	0.99915	0.99732	0.00183
4	2	1.00000	0.99973	0.00027
5	0	1.00000	0.99997	0.00003
6	0	1.00000	1.00000	0.00000
7	0	1.00000	1.00000	0.00000
8	0	1.00000	1.00000	0.00000
9	0	1.00000	1.00000	0.00000
10	0	1.00000	1.00000	0.00000
11	0	1.00000	1.00000	0.00000
12	0	1.00000	1.00000	0.00000
13	0	1.00000	1.00000	0.00000
14	0	1.00000	1.00000	0.00000
15	0	1.00000	1.00000	0.00000
16	0	1.00000	1.00000	0.00000
17	0	1.00000	1.00000	0.00000
18	0	1.00000	1.00000	0.00000
19	0	1.00000	1.00000	0.00000
20	0	1.00000	1.00000	0.00000
OVER 20	0	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 2366

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 1.750   AUTOCORRELATION PARAMETER = 2.500

OBSERVED MEAN = 1.05657   OBSERVED VARIANCE = 0.05539
PREDICTED MEAN = 1.03711   PREDICTED VARIANCE = 0.08885

GAMMA DIST PARAMETERS: ALPHA = 3.24706   BETA = 6.04538

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	935	0.94444	0.92290	0.02154
2	54	0.99899	0.99928	-0.00029
3	1	1.00000	1.00000	0.00000
4	0	1.00000	1.00000	0.00000
5	0	1.00000	1.00000	0.00000
6	0	1.00000	1.00000	0.00000
7	0	1.00000	1.00000	0.00000
8	0	1.00000	1.00000	0.00000
9	0	1.00000	1.00000	0.00000
10	0	1.00000	1.00000	0.00000
11	0	1.00000	1.00000	0.00000
12	0	1.00000	1.00000	0.00000
13	0	1.00000	1.00000	0.00000
14	0	1.00000	1.00000	0.00000
15	0	1.00000	1.00000	0.00000
16	0	1.00000	1.00000	0.00000
17	0	1.00000	1.00000	0.00000
18	0	1.00000	1.00000	0.00000
19	0	1.00000	1.00000	0.00000
20	0	1.00000	1.00000	0.00000
OVER 20	0	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 990

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 2.500   AUTOCORRELATION PARAMETER = 2.500

OBSERVED MEAN = 1.01527   OBSERVED VARIANCE = 0.01504
PREDICTED MEAN = 0.98398   PREDICTED VARIANCE = 0.00719

GAMMA DIST PARAMETERS: ALPHA = 32.56425   BETA = 67.28482

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	129	0.98473	1.00000	-0.01527
2	2	1.00000	1.00000	0.00000
3	0	1.00000	1.00000	0.00000
4	0	1.00000	1.00000	0.00000
5	0	1.00000	1.00000	0.00000
6	0	1.00000	1.00000	0.00000
7	0	1.00000	1.00000	0.00000
8	0	1.00000	1.00000	0.00000
9	0	1.00000	1.00000	0.00000
10	0	1.00000	1.00000	0.00000
11	0	1.00000	1.00000	0.00000
12	0	1.00000	1.00000	0.00000
13	0	1.00000	1.00000	0.00000
14	0	1.00000	1.00000	0.00000
15	0	1.00000	1.00000	0.00000
16	0	1.00000	1.00000	0.00000
17	0	1.00000	1.00000	0.00000
18	0	1.00000	1.00000	0.00000
19	0	1.00000	1.00000	0.00000
20	0	1.00000	1.00000	0.00000
OVER 20	0	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 131

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 0.000   AUTOCORRELATION PARAMETER = 5.000

OBSERVED MEAN = 1.94846   OBSERVED VARIANCE = 1.76810
PREDICTED MEAN = 1.88194   PREDICTED VARIANCE = 1.72155

GAMMA DIST PARAMETERS: ALPHA = 1.10933   BETA = 0.80273

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	3218	0.50877	0.50156	0.00722
2	1588	0.75984	0.76606	-0.00622
3	791	0.88490	0.89180	-0.00690
4	384	0.94561	0.95033	-0.00471
5	180	0.97407	0.97730	-0.00323
6	87	0.98783	0.98966	-0.00183
7	37	0.99368	0.99530	-0.00162
8	26	0.99779	0.99787	-0.00008
9	10	0.99937	0.99903	0.00033
10	3	0.99984	0.99956	0.00028
11	0	0.99984	0.99980	0.00004
12	1	1.00000	0.99991	0.00009
13	0	1.00000	0.99996	0.00004
14	0	1.00000	0.99998	0.00002
15	0	1.00000	0.99999	0.00001
16	0	1.00000	1.00000	0.00000
17	0	1.00000	1.00000	0.00000
18	0	1.00000	1.00000	0.00000
19	0	1.00000	1.00000	0.00000
20	0	1.00000	1.00000	0.00000
OVER 20	0	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 6325

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 0.750 AUTOCORRELATION PARAMETER = 5.000

OBSERVED MEAN = 1.29446 OBSERVED VARIANCE = 0.37522
PREDICTED MEAN = 1.33485 PREDICTED VARIANCE = 0.42310

GAMMA DIST PARAMETERS: ALPHA = 1.64730 BETA = 1.97316

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	3348	0.77019	0.69057	0.07961
2	786	0.95100	0.93979	0.01121
3	159	0.98758	0.98958	-0.00200
4	41	0.99701	0.99829	-0.00128
5	12	0.99977	0.99973	0.00004
6	1	1.00000	0.99996	0.00004
7	0	1.00000	0.99999	0.00001
8	0	1.00000	1.00000	0.00000
9	0	1.00000	1.00000	0.00000
10	0	1.00000	1.00000	0.00000
11	0	1.00000	1.00000	0.00000
12	0	1.00000	1.00000	0.00000
13	0	1.00000	1.00000	0.00000
14	0	1.00000	1.00000	0.00000
15	0	1.00000	1.00000	0.00000
16	0	1.00000	1.00000	0.00000
17	0	1.00000	1.00000	0.00000
18	0	1.00000	1.00000	0.00000
19	0	1.00000	1.00000	0.00000
20	0	1.00000	1.00000	0.00000
OVER 20	0	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 4347

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 1.250 AUTOCORRELATION PARAMETER = 5.000

OBSERVED MEAN = 1.12570 OBSERVED VARIANCE = 0.13993
PREDICTED MEAN = 1.14004 PREDICTED VARIANCE = 0.11490

GAMMA DIST PARAMETERS: ALPHA = 3.56518 BETA = 5.57021

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	2066	0.88631	0.86011	0.02621
2	242	0.99013	0.99751	-0.00738
3	20	0.99871	0.99998	-0.00126
4	1	0.99914	1.00000	-0.00086
5	2	1.00000	1.00000	0.00000
6	0	1.00000	1.00000	0.00000
7	0	1.00000	1.00000	0.00000
8	0	1.00000	1.00000	0.00000
9	0	1.00000	1.00000	0.00000
10	0	1.00000	1.00000	0.00000
11	0	1.00000	1.00000	0.00000
12	0	1.00000	1.00000	0.00000
13	0	1.00000	1.00000	0.00000
14	0	1.00000	1.00000	0.00000
15	0	1.00000	1.00000	0.00000
16	0	1.00000	1.00000	0.00000
17	0	1.00000	1.00000	0.00000
18	0	1.00000	1.00000	0.00000
19	0	1.00000	1.00000	0.00000
20	0	1.00000	1.00000	0.00000
OVER 20	0	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 2331

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 1.750   AUTOCORRELATION PARAMETER = 5.000

OBSERVED MEAN = 1.03747   OBSERVED VARIANCE = 0.03821
PREDICTED MEAN = 1.03076   PREDICTED VARIANCE = 0.01720

GAMMA DIST PARAMETERS: ALPHA = 16.37617   BETA = 30.85400

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	900	0.96360	0.99837	-0.03478
2	33	0.99893	1.00000	-0.00107
3	1	1.00000	1.00000	0.00000
4	0	1.00000	1.00000	0.00000
5	0	1.00000	1.00000	0.00000
6	0	1.00000	1.00000	0.00000
7	0	1.00000	1.00000	0.00000
8	0	1.00000	1.00000	0.00000
9	0	1.00000	1.00000	0.00000
10	0	1.00000	1.00000	0.00000
11	0	1.00000	1.00000	0.00000
12	0	1.00000	1.00000	0.00000
13	0	1.00000	1.00000	0.00000
14	0	1.00000	1.00000	0.00000
15	0	1.00000	1.00000	0.00000
16	0	1.00000	1.00000	0.00000
17	0	1.00000	1.00000	0.00000
18	0	1.00000	1.00000	0.00000
19	0	1.00000	1.00000	0.00000
20	0	1.00000	1.00000	0.00000
OVER 20	0	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 934

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
USING PREDICTED VALUES

A-LEVEL = 2.500 AUTOCORRELATION PARAMETER = 5.000

OBSERVED MEAN = 1.00704 OBSERVED VARIANCE = 0.00699
PREDICTED MEAN = 0.98610 PREDICTED VARIANCE = 0.00019

GAMMA DIST PARAMETERS: ALPHA =1238.77140 BETA =2548.39640

DURATION	NUMBER OBSERVED	CUMULATIVE OBSERVED	CUMULATIVE EXPECTED	DIFFERENCE
1	141	0.99296	1.00000	-0.00704
2	1	1.00000	1.00000	0.00000
3	0	1.00000	1.00000	0.00000
4	0	1.00000	1.00000	0.00000
5	0	1.00000	1.00000	0.00000
6	0	1.00000	1.00000	0.00000
7	0	1.00000	1.00000	0.00000
8	0	1.00000	1.00000	0.00000
9	0	1.00000	1.00000	0.00000
10	0	1.00000	1.00000	0.00000
11	0	1.00000	1.00000	0.00000
12	0	1.00000	1.00000	0.00000
13	0	1.00000	1.00000	0.00000
14	0	1.00000	1.00000	0.00000
15	0	1.00000	1.00000	0.00000
16	0	1.00000	1.00000	0.00000
17	0	1.00000	1.00000	0.00000
18	0	1.00000	1.00000	0.00000
19	0	1.00000	1.00000	0.00000
20	0	1.00000	1.00000	0.00000
OVER 20	0	1.00000	1.00000	0.00000

TOTAL NUMBER OF OVERSHOOTS = 142

APPENDIX C

PREDICTION PROGRAM

IBM

INFORMATION RECORDS DIVISION
MULTIPLE-CARD LAYOUT FORM

IBM H47090
Printed in U.S.A.

Company ______

Application CARD INPUT FORMATS by ______ Date ______ Job No. ______ Sheet No. ______

	PROCESS MEAN (F10.3)	PROCESS STANDARD DEVIATION (F10.3)	AUTO-CORRELATION PARAMETER (F10.3)	CROSSING LEVEL (F10.3)	DURATION (F10.3)	
	1–10	11–20	21–30	31–40	41–50	51–80

```
C PROGRAM PURPOSE : FOR A GIVEN PROCESS, CALCULATE THE PROBABILITY OF
C                   EXCEEDING A SELECTED CROSSING LEVEL FOR X TIME UNITS
C
C REQUIRED INPUT  : 1) PROCESS MEAN, MU
C                   2) PROCESS STANDARD DEVIATION, STDEV
C                   3) AUTOCORRELATION PARAMETER, BETA
C                   4) CROSSING LEVEL, L
C                   5) DURATION, X
C
      REAL MU,L
      INTEGER C,P
C
C I/O ASSIGNMENTS
      C = 5
      P = 6
C
C INPUT PARAMETERS
      READ(C,90) MU,STDEV,BETA,L,X
      SIGMA = STDEV**2
C
C CALCULATE ADJUSTED CROSSING LEVEL
      ALVL = (L-MU)/STDEV
      ALVL = ABS(ALVL)
C
C CALCULATE COEFFICIENTS
      R = ALOG(BETA)
      A0 =  0.92590 - 0.31129*R + 0.06417*R**2 + 0.00988*R**3
      A1 = -0.64060 + 0.02881*R + 0.02002*R**2 - 0.00026*R**3
      A2 =  0.11763 + 0.01749*R - 0.01017*R**2 - 0.00131*R**3
      B0 =  1.88589 - 1.02364*R + 0.56796*R**2 - 0.09535*R**3
      B1 = -1.17242 + 0.74806*R - 0.38702*R**2 - 0.01600*R**3
      B2 =  0.20194 - 0.16723*R + 0.07649*R**2 + 0.01833*R**3
C
C CALCULATE PREDICTED MEAN AND VARIANCE
      R = A0 + A1*ALVL + A2*ALVL**2
      PMEAN = EXP(R)
      R = B0 + B1*ALVL + B2*ALVL**2
      PVAR = R**2
C
C CALCULATE PROBABILITY OF EXCEEDING L FOR X TIME UNITS
      PROB = 1. - GAMX(X,PMEAN,PVAR)
C
C OUTPUT ROUTINE
      WRITE(P,95) BETA,L,ALVL,MU,PMEAN,SIGMA,PVAR,X,PROB
C
C FORMATS
   90 FORMAT(5F10.3)
   95 FORMAT('1',40X,'AUTOCORRELATION PARAMETER =',F11.3//41X,'CROSSING
     1LEVEL    =',F11.3,'    ADJUSTED LEVEL            =',F11.3//41X,'PROCESS
     2MEAN      =',F11.3,'    DISTRIBUTION MEAN       =',F11.3//41X,'PROCESS
     3 VARIANCE =',F11.3,'    DISTRIBUTION VARIANCE =',F11.3///50X,'PROBA
     4BILITY OF EXCEEDING THE CROSSING LEVEL FOR'/56X,F11.3,' TIME UNITS
     5 =',F6.3)
C
```

STOP
END

```
C     DOUBLE PRECISION INCOMPLETE GAMMA, GAMX (X,DF), -- PUNCHED ON 026
      DOUBLE PRECISION FUNCTION GAMX(T,XBAR,VAR)
      IMPLICIT  REAL*8(D)
      DOUBLE PRECISION YORMX
      DE = XBAR**2./VAR
      DX = (T*XBAR)/VAR
      DY = DX
      DF = DE
      DSUM = 0.D0
      IF(DX.GT.0.D0) GO TO 2
      GAMX = 0.D0
      GO TO 99
 2    IF(DF.GT.0.D0) GO TO 4
      GAMX = 1.D0
      GO TO 99
 4    IF((DF.GE.200.D0).AND.(DY.LE.DF)) GO TO 21
      IF((DF.GE.2.D0).AND.(DY.GE.DF+3.D0*DSQRT(DF)))  GO TO 40
      DAI = DF
      DDF = DAI*DLOG(DY) - DY - DLGGM(DAI + 1.D0)
 16   IF(DDF.LE.-80.D0) GO TO 10
      DFG = DEXP(DDF)
 12   DFH = DFG
      DSUM = DSUM + DFG
      DFG = DFG*DY/(DAI + 1.D0)
      DAI = DAI + 1.D0
      IF(DAI.GT.200.D0) GO TO 25
      IF(DFG.LT.DFH) GO TO 13
      IF(DAI.GT.200.D0) GO TO 25
      GO TO 12
 13   DFH = DFG
      IF(DFG/DSUM.LE.1.D-14) GO TO 15
      GO TO 12
 10   DAI = DAI + 1.D0
      IF(DAI.GT.200.D0) GO TO 25
      DDF = DDF + DLOG(DY/DAI)
      GO TO 16
 21   DH = 9.D0*DF
 26   DYN = ((DY/DF)**0.333333333333333 - 1.D0 + 1.D0/DH)*DSQRT(DH)
      DNMIX = YORMX(DYN)
      GAMX = DNMIX + DSUM
      GO TO 99
 15   GAMX = DSUM
      GO TO 99
 25   DH = 9.D0*DAI
      DF = DAI
      GO TO 26
 40   DFX = DF*DLOG(DY) - DY - DLGGM(DF)
      DAL = 0.D0
      DAH = 1.D0
      DBL = 1.D0
      DBH = DY
      DBK = 1.D0
      DBKP = DY
      DK = 1.D0
```

```
42    DAK = DK - DF
      DAKP = DK
      DAL = DBK*DAH + DAK*DAL
      DBL = DBK*DBH + DAK*DBL
      DAH = DBKP*DAL + DAKP*DAH
      DBH = DBKP*DBL + DAKP*DBH
      DFA = DAL/DBL
      DFB = DAH/DBH
      IF(DFB.EQ.0.D0) GO TO 45
      IF(DABS((DFA-DFB)/DFB).LE.1.D-13) GO TO 41
      DK = DK + 1.D0
      GO TO 42
41    DFX = DFX + DLOG(DFB)
      GAMX = 1.D0
      IF(DFX.GE.-80.D0)  GAMX = 1.D0 - DEXP(DFX)
      GO TO 99
45    GAMX = 1.D0
      GO TO 99
99    RETURN
      END
```

```
      DOUBLE PRECISION FUNCTION YORMX(DZ)
      IMPLICIT REAL*8(D)
      DPI = .398942280401433
      DX = DABS(DZ)
      IF(DX.GT.3.D0) GO TO 10
      DAL = 0.D0
      DBL = 1.D0
      DAH = DX
      DBH = 1.D0
      DAN = 0.D0
    5 DAN = DAN + 1.D0
      DAI = -(2.D0*DAN - 1.D0) *DX*DX
      DBI = 4.D0*DAN - 1.D0
      DAL = DBI*DAH + DAI*DAL
      DBL = DBI*DBH + DAI*DBL
      DAI = DX*DX - DAI
      DBI = 2.D0 + DBI
      DAH = DBI*DAL + DAI*DAH
      DBH = DBI*DBL + DAI*DBH
      DFA = DAL/DBL
      DFB = DAH/DBH
      IF(DFB.EQ.0.D0) GO TO 20
      IF(DABS((DFB-DFA)/DFA).LE.1.D-14) GO TO 20
      GO TO 5
   10 DAL = 0.D0
      DBL = 1.D0
      DAH = 1.D0
      DBH = DX
      DBI = DX
      DAN = 1.D0
      DFA = 1.D0/DX
   15 DAN = DAN + 1.D0
      DAI = DAN - 1.D0
      DAC = DBI*DAH + DAI*DAL
      DBC = DBI*DBH + DAI*DBL
      DFB = DAC/DBC
      DAL = DAH
      DBL = DBH
      DAH = DAC
      DBH = DBC
      IF(DFB.EQ.0.D0) GO TO 20
      IF(DABS((DFB-DFA)/DFB).LE.1.D-14) GO TO 20
      DFA = DFB
      GO TO 15
   20 YORMX = DPI*DFB*DEXP(-DX*DX/2.D0)
      IF(DX.LE.3.D0) YORMX = 0.5D0 - YORMX
      IF(DZ.GT.0.D0) YORMX = 1.D0 - YORMX
      RETURN
      END
```

```
      DOUBLE PRECISION FUNCTION DLGGM(DX)
      IMPLICIT REAL*8(D)
      DY=DX
      DTERM=1.D0
      IF(DX)1,1,2
    1 DLGGM=0.D0
      RETURN
    2 IF(DY-18.D0) 3,3,4
    3 DTERM=DTERM*DY
      DY=DY+1.D0
      GO TO 2
    4 DLGGM= (DY-.5D0)* DLOG(DY)-DY+1.D0/(12.D0*DY)-1.D0/(360.D0*DY**3)
     1 +1.D0/ (1260.D0*DY**5) -1.D0/ (1680.D0*DY**7) +.918938533204673D0
     2-DLOG(DTERM)
      RETURN
      END
```